Coordinate Systems And Time

Learners' Space Astronomy

Contents

Introduction

In this module, we will study astronomical coordinate systems, directions and apparent motions of celestial objects, determination of position from astronomical observations, observational errors, astronomical time systems etc. We shall concentrate mainly on different astronomical coordinate systems, sidereal and synodic time, analemma and different time systems. For simplicity, we will assume that the observer is always in the northern hemisphere. Although all definitions and equations are easily generalized for both hemispheres, this might be unnecessarily confusing. In spherical astronomy all angles are usually expressed in degrees; we will also use degrees unless otherwise mentioned.

1.1 Geometry of Sphere

A **great circle** is defined to be the intersection with the sphere of a plane containing the centre of the sphere.

There is exactly one great circle passing through two given points Q and Q' on a sphere (unless these points are diametrically opposite (antipodal), in which case all circles passing through both of them are great circles). The arc QQ' of this great circle is the shortest path on the surface of the sphere between these points.

If the plane does not contain the centre of the sphere, its intersection with the sphere is a **small circle**.

A line perpendicular to the plane of a great circle and passing through the centre of the sphere intersects the sphere at the **poles** P and P'.

If three great circles intersect one another so that a closed figure is formed by three arcs of the great circles, it is called a **spherical triangle**.

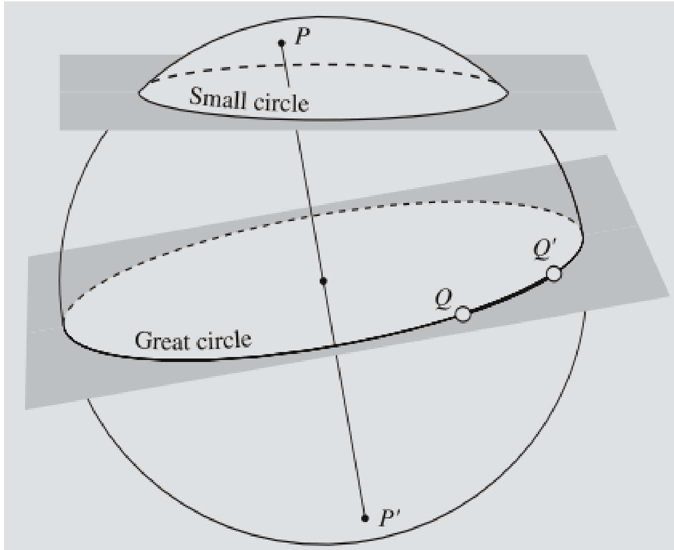

Figure 1.1: The above figure shows the difference between a small circle and a great circle

The spherical triangle ABC has the arcs AB, BC and AC as its sides. If the radius of the sphere is r, the arc length AB is $|AB| = rc$, where c is the angle subtended (in radians) by the arc AB as seen from the centre. This angle is called the central angle of the side AB. Because side lengths and central angles uniquely correspond to each other, it is customary to give the central angles instead of the sides. In this way, the radius of the sphere does not enter into the equations of spherical trigonometry.

The angle between the tangents at A to the great circles containing sides AC and AB or equivalently the dihedral angle between the planes of those great circles is said to be the **spherical angle** at A.

We denote the angles of a spherical triangle by capital letters (A, B, C) and the opposing side lengths, or, more correctly, the corresponding central angles, by lowercase letters (a, b, c)

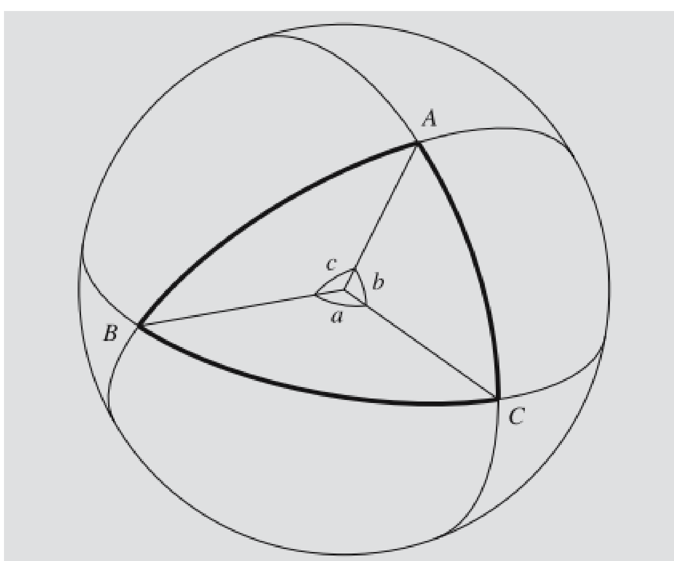

Figure 1.2: The angles A, B, C are spherical angles of the triangle while a,b,c are corresponding central angles

1.2 Formulae of Spherical Trigonometry

As in plane trigonometry, we have sin and cosine laws in spherical trigonometry which indeed are a much more general form of these laws.
Sine Rule:

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

Cosine Rule:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

Analogue of Cosine Rule:

$$\cos B \sin a = -\cos A \sin b cos c + \cos b \sin c$$

Four parts formula:

$$\cos a \cos C = \sin a \cot b - \sin C \cot B$$

This formula utilizes four consecutive parts of the spherical triangle and is often stated as follows:

$$\cos(innerside) \cos(innerangle) = \sin(innerside) \cot(otherside) - sin(innerangle) \cot(otherangle)$$

Equations for other sides and angles are obtained by cyclic permutations of the sides a, b, c and the angles A, B, C.

Corresponding to every spherical triangle ΔABC, there is a polar triangle $\Delta A'B'C'$ defined as follows: Consider the great circle that contains the side BC. This great circle has two poles. The pole that lies on the same side of the great circle's plane as A is denoted by A'. The points B' and C' are defined similarly. It can be shown that the angles and sides of the polar triangle are given by:

$$A' = \pi - a, \quad B' = \pi - b, \quad C' = \pi - c$$
$$a' = \pi - A, \quad b' = \pi - B, \quad c' = \pi - C$$

Therefore, if any identity is proved for Δ ABC then we can immediately derive a second identity by applying the first identity to the polar triangle and making the above substitutions.

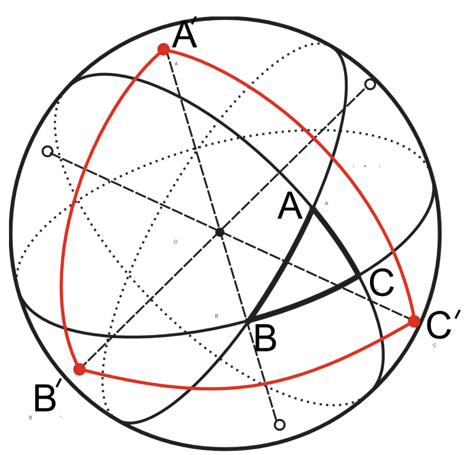

Figure 1.3: The polar triangle $\Delta A'B'C'$

The Celestial Sphere

The ancient folks had a very simplistic view of the universe. They thought the universe to be confined within a finite spherical shell. The stars were fixed to this shell while some objects, called wanderers(planets) moved on this sphere and thus they were all equidistant from the earth, which was at the centre of the spherical universe. This simple model is still in many ways as useful as it was in antiquity: it helps us to easily understand the diurnal and annual motions of stars, and, more importantly, to predict these motions in a relatively simple way.

Therefore we will assume for the time being that all the stars are located on the surface of an enormous sphere and that we are at its centre. Because the radius of this celestial sphere is practically infinite, we can neglect the effects due to the changing position of the observer, caused by the rotation and orbital motion of the earth.

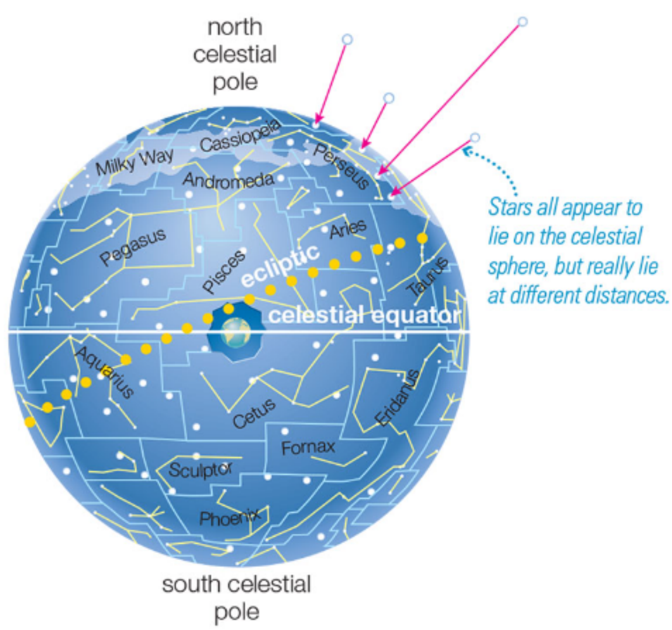

Figure 2.1: The Celestial Sphere, depicting North Celestial Pole, South Celestial Pole, ecliptic and celestial equator

Since the distances of the stars are ignored, we need only two coordinates to specify their directions. This is just like the system of latitudes and longitudes on the earth. Each coordinate system has some fixed reference plane passing through the centre of the celestial sphere and dividing the sphere into two hemispheres along a great circle. One of the coordinates indicates the angular distance from this reference plane. There is exactly one great circle going through the object and intersecting this plane perpendicularly; the second coordinate gives the angle between that point of intersection and some fixed direction.

For example, look at the latitudes and longitudes on the Earth; when we try to specify the coordinates of a place we define latitude as the angular distance between that place and the equator plane (fixed reference plane) and for defining longitude we take the great circle passing through the place and perpendicular to the equator plane and measure its angular distance from the Greenwich meridian (fixed direction).

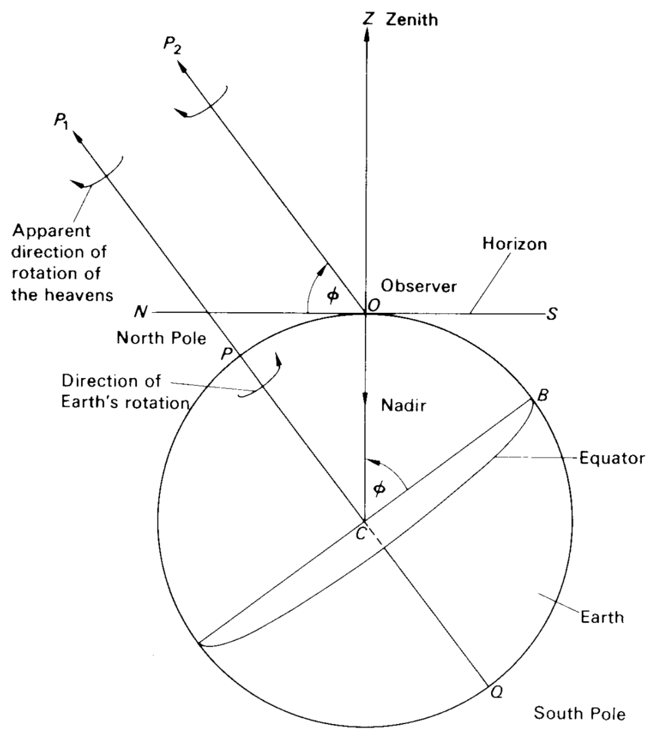

Figure 2.2: Definitions related to the observer's position on the Earth.

Because of the Earth's rotation about its north-south axis PQ (see Figure 2.2), the heavens appear to revolve in the opposite direction about a point P_1 which is the intersection of QP with the celestial sphere. Because the radius of the sphere is infinite compared with the radius of the Earth, this point is indistinguishable from a point P_2, where OP_2 is parallel to QPP_1. P_2 is then said to be the **north celestial pole** and all stars trace out circles of various sizes centred on P_2. Even Polaris, the Pole Star, is about one degree from the pole-a large angular distance astronomically speaking when we remember that four full moons (the angular diameter of the Moon is $30'$) could be laid side-by-side within Polaris' circular path about the north celestial pole.

The points N and S are the points where the great circle from the zenith through the north celestial pole, called the **local meridian**, meets the horizon, the north point, N, being the nearer of the two to the pole.

The Horizontal System

The most natural coordinate system from the observer's point of view is the horizontal system (Fig. 3.1). Its reference plane is the tangent plane of the earth passing through the observer; this horizontal plane intersects the celestial sphere along the great circle called horizon. The point just above the observer is called the **zenith** and the antipodal point below the observer is the **nadir**. (These two points are the poles corresponding to the horizon.) Great circles through the zenith are called **verticals**. All verticals intersect the horizon perpendicularly.

By observing the motion of a star over the course of a night, an observer finds out that it follows a track like one of those in Fig. 3.2 . Stars rise in the east, reach their highest point, or **culminate**, on the vertical NZS (North, Zenith, South), and set in the west. The vertical NZS is also called the **local meridian**, so we can define **culmination** of a star as the time when it is on the local meridian.

One of the horizontal coordinates is the **altitude** (or elevation) denoted as 'a', which is measured from the horizon along the vertical passing through the object. The altitude lies in the range $[-90°, +90°]$; it is positive for objects above the horizon and negative for objects below the horizon. The zenith distance, or the angle between the object and the zenith, is then

$$z = 90° - a$$

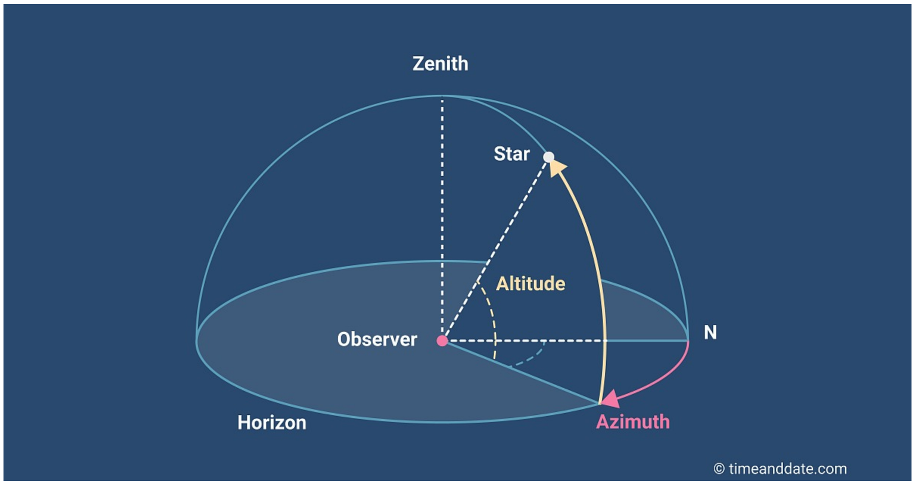

Figure 3.1: The altitude and azimuth of a star in the horizontal system

The second coordinate is the **azimuth**, A; it is the angular distance of the vertical of the object from some fixed direction. Unfortunately, in different contexts, different fixed directions are used; thus it is always advisable to check which definition is employed. Commonly, it is measured from the north point eastwards (clockwise) from 0° to 360°.

Since the horizontal or alt-az coordinates are time and position-dependent, they cannot be used, for instance, in star catalogues. So we need a uniform system in which a star's coordinates are independent of the time and position of the observer on Earth which brings us to the Equatorial System.

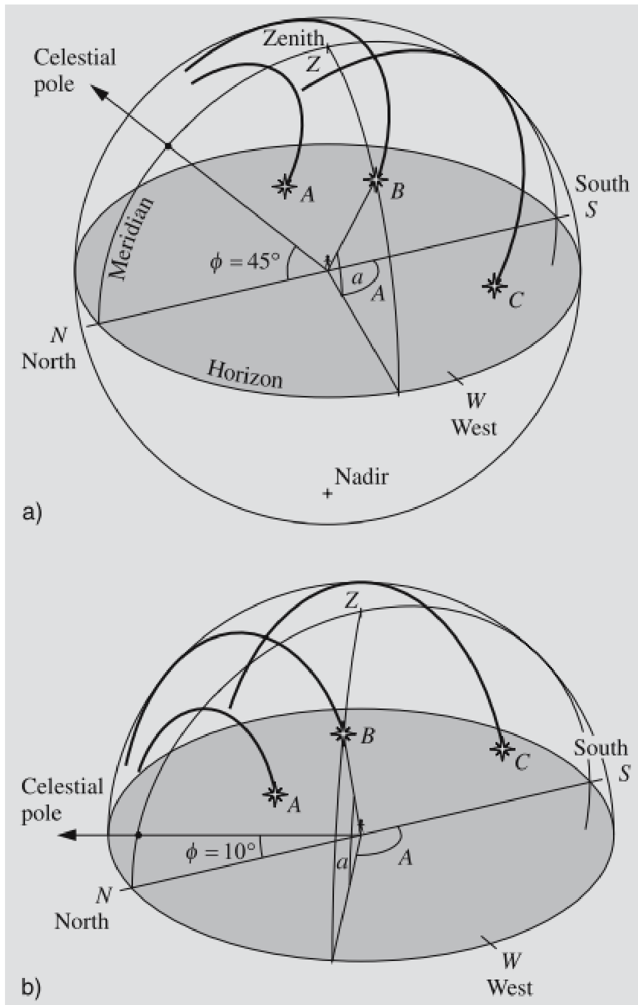

Figure 3.2: (a) The apparent motions of stars during a night as seen from latitude $\phi = 45°$. (b) The same stars seen from latitude $\phi = 10°$

The Equatorial System

The direction of the rotation axis of the earth remains almost constant and so does the equatorial plane perpendicular to this axis. Therefore the equatorial plane is a suitable reference plane for a coordinate system that has to be independent of time and the position of the observer.

The intersection of the celestial sphere and the equatorial plane is a great circle, which is called the **equator** of the celestial sphere. The angular separation of a star from the equatorial plane is not affected by the rotation of the Earth. This angle is called the **declination** δ.

There is one more significant great circle which is called the **ecliptic**. This represents the path of the sun on the celestial sphere over the course of the year. The ecliptic passes through twelve constellations on the celestial sphere, and these constellations are called **zodiac constellations**. They are (in order): Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpius, Sagittarius, Capricornus, Aquarius and Pisces. The points of intersection of the ecliptic and celestial equator are called equinoxes (equal day and night). The one occurring when the sun moves from the southern hemisphere to the northern hemisphere is called **vernal equinox**(usually around 21^{st} March) and the other one (when the sun moves from north to south) is called **autumnal equinox**(close to 21^{st} September). The time of the year when the sun attains its maximum declination is called **Summer Solstice**(21^{st} June). Likewise, **Winter Solstice**(22^{nd} December) occurs at the time of minimum declination of the sun.

The point where the extension of the earth's rotational axis meets the celestial sphere is called **North Celestial pole(NCP)** if it lies to the north of the equator and similarly **South Celestial pole(SCP)** on its other end. Apart from these two poles, we have other two also corresponding to the ecliptic, these are **North Ecliptic pole(NEP)** and **South Ecliptic pole(SEP)**, though these are of less importance. The NCP is at a distance of about one degree (which is equivalent to two full moons) from the moderately bright star *Polaris*. The **local meridian** as discussed earlier is the semicircle in the sky joining NCP, zenith and SCP. It is always perpendicular to the horizon.

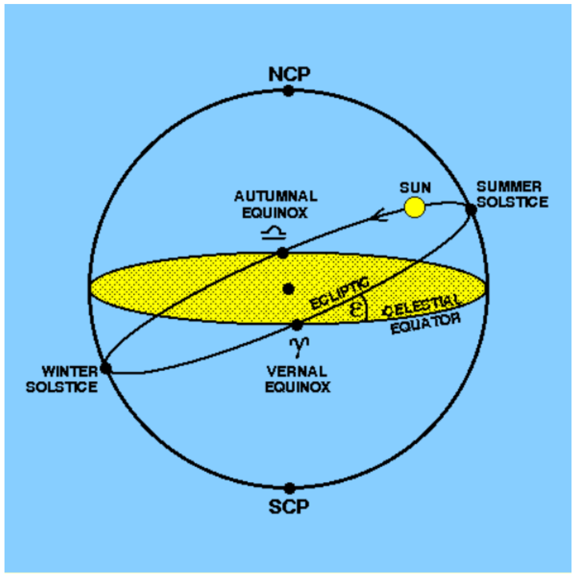

Figure 4.1: Ecliptic on the celestial sphere with vernal and autumnal equinox.

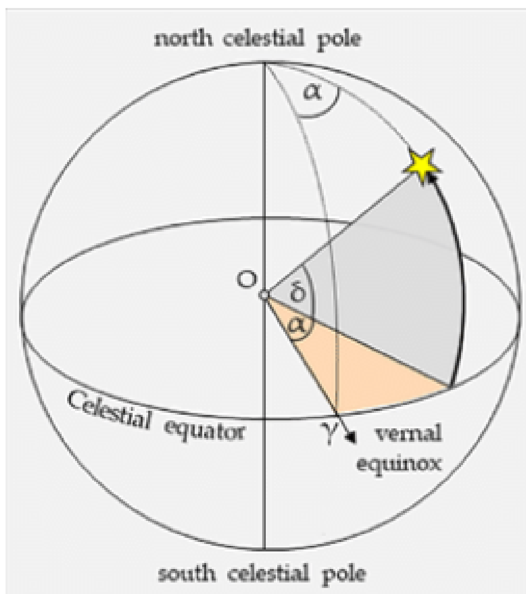

Figure 4.2: The declination and right ascension of a star on the celestial sphere

To define the second coordinate, we must again agree on a fixed direction, unaffected by the earth's rotation. From a mathematical point of view, it does not matter which point on the equator is selected. However, it is more appropriate to employ a certain point with some valuable properties, like the point of intersection of the celestial equator and ecliptic. This point is the **vernal equinox**. Because it used to be in the constellation Aries (the Ram), it is also called the **first point of Aries** and denoted by the sign of Aries Υ.

Now we can define the second coordinate as the angle from the vernal equinox measured along the equator. This angle is the **right ascension** α (or RA) of the object, measured counterclockwise (eastwards) from Υ.

Even the position of vernal equinox is not fixed!

We know that the earth undergoes rotation about its axis and revolution around the sun. Apart from these two movements, the earth also shows precession. Precession is the change in the orientation of the earth's rotation axis. This is similar to what we see in a toy top. The top revolves around its axis and its axis in turn revolves around another axis, tracing out a cone.
Video explanation(animation): https://www.youtube.com/watch?v=qlVgEoZDjok
This is really slow, but due to this, the vernal equinox, which was earlier defined in Aries has not shifted to Pisces. One cycle of precession takes around 26,000 years.
Due to precession, Vega (a bright star in Lyra) will be our pole star in the next 14,000 years!

Since declination and right ascension are independent of the position of the observer and the motions of the earth, they can be used in star maps and catalogues.

But can we use these coordinates directly to find an object in the night sky? As you will see later in the module on Telescopes, commonly used *equatorial mount telescopes* have one of their axes (the hour axis) parallel to the rotation axis of the Earth and the other axis (declination axis) perpendicular to the hour axis. Declinations can be read immediately on the declination dial of the telescope. But the zero point of the right ascension seems to move in the sky, due to the diurnal rotation of the earth. So we cannot use the right ascension to find an object unless we know the direction of the vernal equinox.

Since the local meridian is a well-defined line in the sky, we use it to establish a local coordinate corresponding to the right ascension. The **hour angle** of a celestial object is the angle it makes from the local meridian measured clockwise (westwards) along the equator, expressed in time units. The hour angle of any object say a star is zero at culmination and grows with time. Hence, it is also the time elapsed since the object's last culmination. The hour angle of the vernal equinox is called the **sidereal time** Θ. Figure 4.3 shows that for any object

$$\Theta = h + \alpha$$

where h is the object's hour angle and α is its right ascension.
This link demonstrates the hour angle, sidereal time and its relation with RA:
https://www.youtube.com/watch?v=1o7T68LZBJI

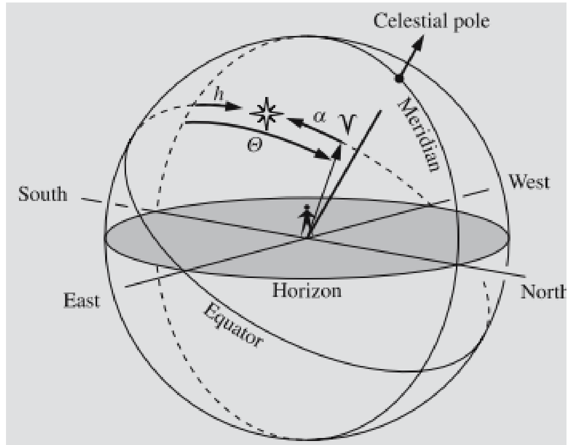

Figure 4.3: The sidereal time Θ (the hour angle of the vernal equinox) equals the hour angle plus right ascension of any object

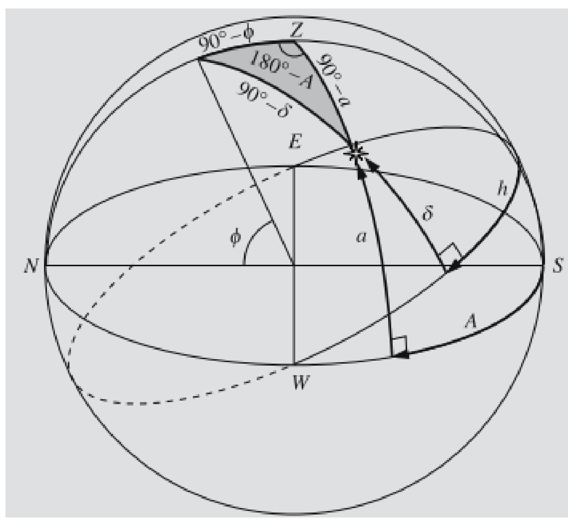

Figure 4.4: The nautical triangle for deriving transformations between the horizontal and equatorial system

Since hour angle and sidereal time change with time at a constant rate, it is practical to express them in units of time. Also, the closely related right ascension is customarily given in time units. Thus 24 hours equals 360 degrees, 1 hour = 15 degrees, 1 minute of time = 15 minutes of arc, and so on. All these quantities are in the range $\left(0^h, 24^h\right)$.

The altitude at the upper culmination (the highest point an object attains during the course of a day) is given by

$$a_{max} = 90^{\circ} - \phi + \delta$$

if the object culminates south of zenith and

$$a_{max} = 90^{\circ} + \phi - \delta$$

if the object culminates north of zenith.

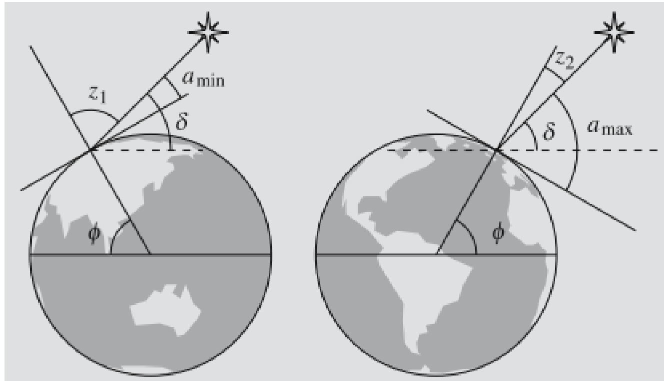

Figure 4.5: The altitude of a circumpolar star at upper and lower culmination

Objects with declinations less than $\phi - 90°$ can never be seen at the latitude ϕ. Similarly, the altitude at the lower culmination is

$$a_{min} = \phi + \delta - 90°$$

Stars with $\delta > 90° - \phi$ will never set (just put $a_{min} > 0$). For example, in Helsinki ($\phi = 60°$), all stars with a declination higher than 30° are circumpolar stars. And stars with a declination less than $-30°$ can never be observed there.
This link illustrates the concept of circumpolar stars:
https://www.youtube.com/watch?v=yMsEjtQ-7n4

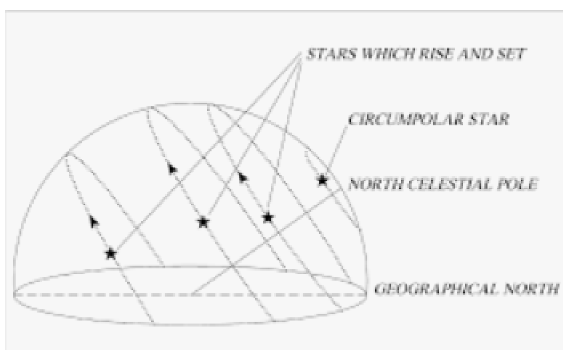

Figure 4.6: The diurnal motion of circumpolar and other stars

We shall now study briefly how the (α, δ) system can be established by observations. Suppose we observe a circumpolar star at its upper and lower culmination (Fig. 4.6). At the upper transit, its altitude is $a_{max} = 90° - \phi + \delta$ and at the lower transit, $a_{min} = \delta + \phi - 90°$. Eliminating the latitude, we get

$$\delta = \frac{1}{2}(a_{min} + a_{max})$$

Thus we get the same value of declination independent of the observer's location as discussed earlier.

Other Coordinate Systems

Apart from the Horizontal and the Equatorial systems, we have two other famous coordinate systems, namely the Ecliptic System and the Galactic System. These are useful in certain settings. We shall just go through the basic definition of both the systems not touching their mathematical part.

5.1 Ecliptic Coordinate System

The orbital plane of the earth, the ecliptic, is the reference plane of another important coordinate system. The ecliptic can also be defined as the great circle on the celestial sphere described by the motion of the sun in the course of one year. The orientation of the earth's equatorial plane remains invariant, unaffected by annual motion.

In spring, the sun appears to move from the southern hemisphere to the northern one (Fig. 5.1). Almost midway between this transition there is a moment called the vernal equinox. At the vernal equinox, the sun's right ascension and declination are zero. The equatorial and ecliptic planes intersect along a straight line directed towards the vernal equinox. Thus we can use this direction as the zero point for both the equatorial and ecliptic coordinate systems. The point opposite the vernal equinox is the autumnal equinox, it is the point at which the sun crosses the equator from north to south.

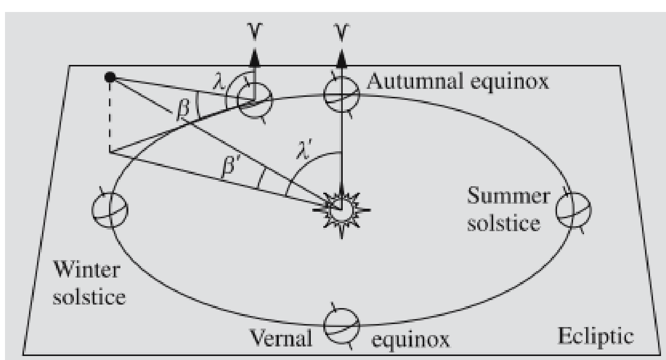

Figure 5.1: The ecliptic geocentric (λ, β) and heliocentric (λ', β') coordinates are equal only if the object is very far away. The geocentric coordinates depend also on the earth's position in its orbit

The ecliptic latitude β is the angular distance from the ecliptic; it is in the range $[-90°, +90°]$.

The other coordinate is the ecliptic longitude λ, measured eastward from the vernal equinox.

Depending on the problem to be solved, we may encounter heliocentric (origin at the sun), geocentric (origin at the centre of the earth) or topocentric (origin at the observer) coordinates. For very distant objects the differences are negligible, but not for bodies of the solar system.

This system is used mainly for planets and other bodies of the solar system. Another use for this coordinate system is in calculating the changes in positions of distant objects due to effects like stellar aberration and parallax, as well as due to the precession of the earth's axis.

5.2 Galactic Coordinate System

For studies of the Milky Way Galaxy, the most natural reference plane is the plane of the Milky Way (Fig. 5.2). Since the sun lies very close to that plane, we can put the origin at the sun. The galactic longitude l is measured counterclockwise (like right ascension) from the direction of the centre of the Milky Way (in Sagittarius, $\alpha = 17^h 45.7^m$, $\delta = -29°$). The galactic latitude b is measured from the galactic plane, positive northwards and negative southwards.

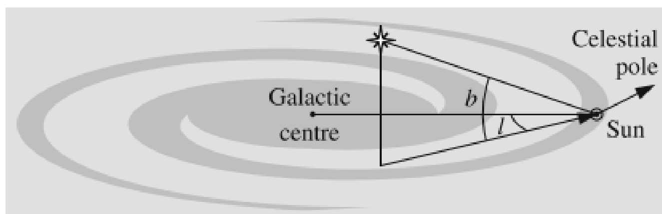

Figure 5.2: The galactic coordinates l and b

Sidereal And Solar Time

Earlier in this chapter we defined the sidereal time as the hour angle of the vernal equinox. A good basic unit is a *sidereal day*, which is the time between two successive upper culminations of the vernal equinox. After one sidereal day, the celestial sphere with all its stars has returned to its original position with respect to the observer. The flow of sidereal time is as constant as the rotation of the earth. This rotation rate is slowly decreasing, due to tidal interactions with the Moon, and thus the length of the sidereal day is increasing. In addition to the smooth slowing down irregular variations of the order of one millisecond have been observed.
Video explanation : https://www.youtube.com/watch?v=xUQKrUpQeXQ

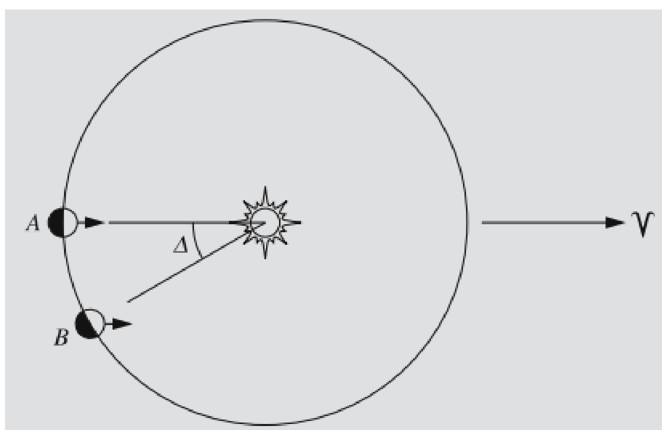

Figure 6.1: One sidereal day is the time between two successive transits or upper culminations of the vernal equinox. By the time the earth has moved from A to B, one sidereal day has elapsed. The angle shown is greatly exaggerated; in reality, it is slightly less than one degree

Figure 6.1 shows the sun and the earth at the vernal equinox. When the earth is at point A, the sun culminates and, at the same time, a new sidereal day begins in the city with the huge black arrow standing in its central square. After one sidereal day, the earth has moved along its orbit almost one degree of arc to point B. Therefore the earth has to turn almost a degree further before the sun will culminate.
The link for animation:
https://www.youtube.com/watch?v=WWw4JY2dNXM&t=3s

The solar or synodic day is therefore 3 min 56.56 s (sidereal time) longer than the sidereal day.

This means that the beginning of the sidereal day will move around the clock during the course of one year. After one year, sidereal and solar time will again be in phase. The number of sidereal days in one year is one more than the number of solar days.

When we talk about rotational periods of planets, we usually mean sidereal periods. The length of day, on the other hand, means the rotation period with respect to the sun. If the orbital period around the sun is P, sidereal rotation period τ^* and synodic day τ, we now know that the number of sidereal days in time P, P/τ^*, is one higher than the number of synodic days, P/τ:

$$\frac{P}{\tau^*} - \frac{P}{\tau} = 1 \tag{6.1}$$

or

$$\frac{1}{\tau} = \frac{1}{\tau^*} - \frac{1}{P} \tag{6.2}$$

This holds for a planet rotating in the direction of its orbital motion (counterclockwise). If the sense of rotation is opposite, or retrograde (e.g. Venus), the number of sidereal days in one orbital period is one less than the number of synodic days, and the equation becomes

$$\frac{1}{\tau} = \frac{1}{\tau^*} + \frac{1}{P} \tag{6.3}$$

For the earth, we have P = 365.2564 d, and τ = 1 d, whence eqn (6.2) gives τ^* = 0.99727 d = 23 h 56 min 4 s, solar time. Since our everyday life follows the alternation of day and night, it is more convenient to base our timekeeping on the apparent motion of the sun rather than that of the stars.

Unfortunately, solar time does not flow at a constant rate. There are two reasons for this. First, the orbit of the earth is not exactly circular, but an ellipse, which means that the velocity of the earth along its orbit is not constant. Second, the sun moves along the ecliptic, not the equator. Thus its right ascension does not increase at a constant rate. The change is fastest at the end of December (4 min 27 s per day) and slowest in mid-September (3 min 35 s per day). As a consequence, the hour angle of the sun (which determines the solar time) also grows at an uneven rate.

To find a solar time flowing at a constant rate, we define a fictitious mean sun, which moves along the celestial equator with constant angular velocity, making a complete revolution in one year. By year we mean here the tropical year, which is the time it takes for the sun to move from one vernal equinox to the next. That is, in one tropical year the right ascension of the sun increases exactly 24 hours. The length of the tropical year is $365^d 5^h 48^m 46^s$ = 365.2422 days.

Since the direction of the vernal equinox moves due to precession, the tropical year differs from the sidereal year, during which the sun makes one revolution with respect to the background stars. One sidereal year is 365.2564 d. Using our artificial mean sun, we now define an evenly flowing solar time, the mean solar time (or simply mean time) T_M, which is equal to the hour angle h_M of the centre of the mean sun plus 12 hours (so that the date will change at midnight, to annoy astronomers):

$$T_M = h_M + 12^h \tag{6.4}$$

The difference between the true solar time T and the mean time T_M is called the equation of time:

$$ET = T - T_M \tag{6.5}$$

The greatest positive value of ET is about 16 minutes and the greatest negative value about -14 minutes.

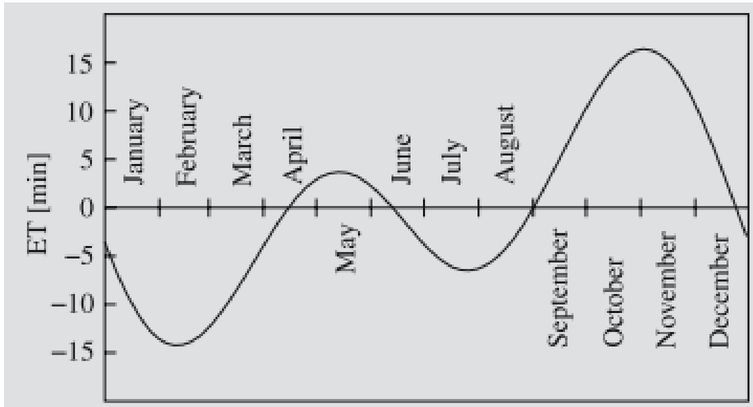

Figure 6.2: Equation of time. A sundial always shows (if correctly installed) true local solar time. To find the local mean time the equation of time must be subtracted from the local solar time

6.1 Analemma

If you looked at the sun at the same (mean solar) time each day, from the same place, would it appear at the same location in the sky? If the earth were not tilted, and if its orbit around the sun were perfectly circular, then, yes, it would. However, a combination of the earth's 23.5°tilt and its slightly elliptical orbit combine to generate this figure "8" pattern of where the sun would appear at the same time throughout the year. This pattern is called an analemma.

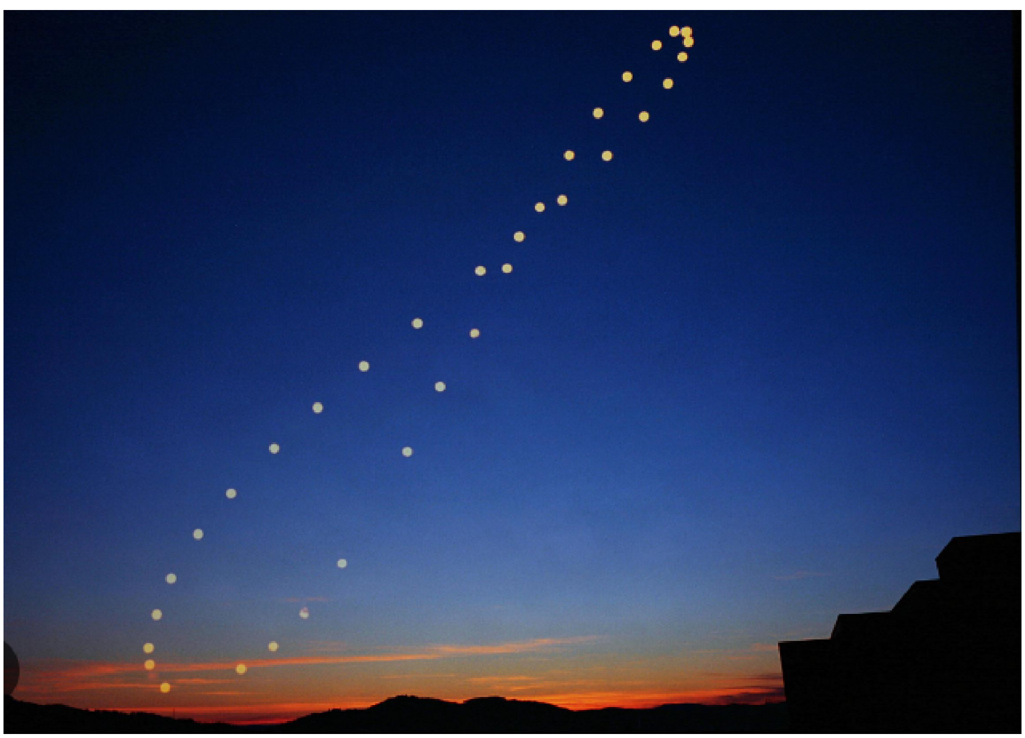

Figure 6.3: An Analemma

The north-south component of the analemma results from the change in the sun's declination due to the tilt of the earth's axis of rotation. The east-west component results from the non-uniform rate of change of the sun's right ascension, governed by the combined effects of the earth's axial tilt and its orbital eccentricity.

The sun will appear at its highest point in the sky, and the highest point in the analemma, during summer. In the winter, the sun is at its lowest point. The in-between times generate the rest of the analemma pattern due to the above-stated reasons. (See Analemma Curve.) Analemmas viewed from different earth latitudes have slightly different orientations but the same shape, as do analemmas created at different times of the day. Analemmas on the other planets have different shapes entirely!

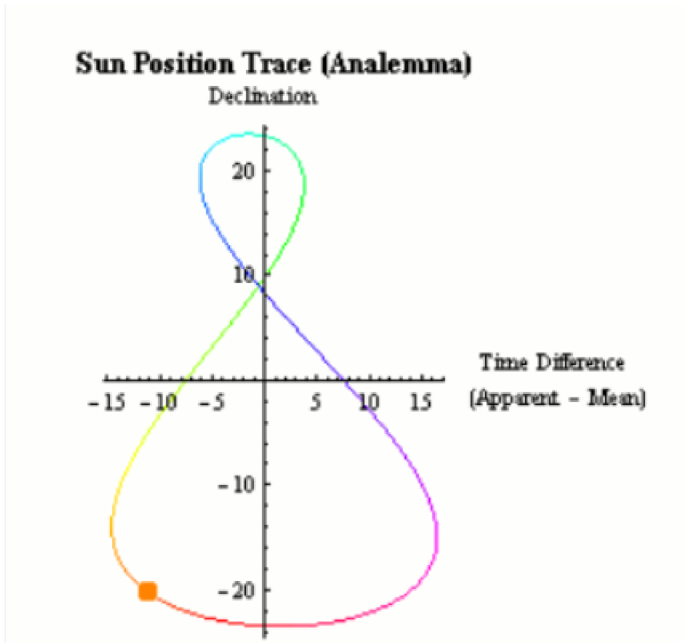

Figure 6.4: The analemma put on Declination vs Equation of Time graph helps us visualize how analemma happens and correlate it with the graph in fig 1.12

It is interesting to note that ET $\neq 0$ at equinoxes. This is because the analemma is a result of the two factors mentioned earlier, for you to recall they were:

1. 23.5° tilt of earth's axis
2. Earth's elliptic orbit

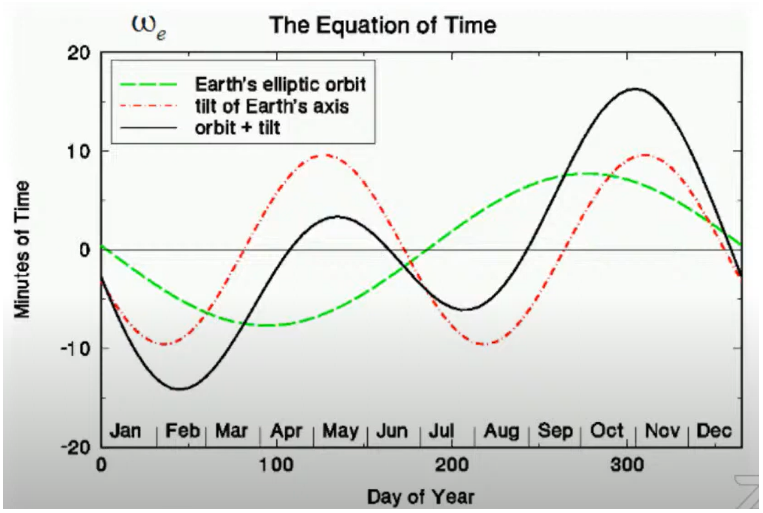

Figure 6.5: The effective graph of ET taking into consideration the two factors mentioned above

Taking into account both of these factors results in a formation of "8" which has a smaller upper lobe as ET is smaller during May-June and a bigger lower lobe because ET has larger value during November-December. Fig 6.5 illustrates it.
This link lets you play around with various parameters for an analemma:
https://www.analemma.com/other-analemmas.html

Months and Years

Just as there are various definitions for day, there are various definitions for month as well as year. Let's start with the month.

7.1 Definitions of Month

Civil month 28 to 31 days
: Month according to the calendar.

Sidereal month $27^d07^h43^m11.6^s$
: One full revolution of the moon around the earth.

Synodic month (aka lunation) $29^d12^h44^m02.9^s$
: One full cycle of lunar phases, i.e., the period between two successive new moons. This differs from the sidereal month due to the revolution of the earth around the sun.

Draconic month $27^d05^h05^m35.8^s$
: Period between successive passes of the moon through the ascending node of its orbit. This differs from the sidereal month due to the precession of the orbit of the moon.

7.2 Definitions of Year

Civil year Various lengths
: Year according to the calendar.

Sidereal year $365^d06^h09^m09.76^s$
: One full revolution of the earth around the sun.

Tropical year $365^d05^h48^m46^s$
: One full cycle of seasons, i.e., the period between two successive vernal equinoxes. This differs from the sidereal year due to the precession of the earth's axis.

Anomalistic year $365^d06^h13^m52.6^s$
: Period between successive passes of the earth through the perihelion of its orbit. This differs from the sidereal month due to the precession of the earth's orbit.

Calendars

Since the Stone Age, as humanity progressed and became civilized, the need for a dating system (calendar dates ;-)) arose. Historians believe that timekeeping dates back to the Neolithic period, but actual calendars were not developed until the Bronze Age in 3100 BC. Early attempts at creating calendars were made by various civilizations, including the Babylonians, Chinese, Egyptians, and others.
These calendars often consisted of 12 months with alternating 29 and 30-day months to align with the moon, resulting in a total of 354 days. However, this was approximately 11 days shorter than a solar year. In this module, we will explore different calendars, such as the Indian (Hindu) calendar, Roman calendar, Julian calendar, and Gregorian calendar. But before that, let's understand the basis of these calendars:

Lunar calendars (e.g. Islamic Hijri calendar)
: In lunar calendars, one month is one full cycle of lunar phases. These calendars start the next year after 12 months, without caring about the cycle of seasons.

Solar calendars (e.g. Julian, Gregorian, Persian, Buddhist, many Indian calendars)
: Solar calendars, on the other hand, aim to have a year consisting of a full cycle of seasons, without caring whether months correspond to lunar cycles.

Lunisolar calendars (e.g. Hindu, Chinese, Hebrew calendars)
: Lunisolar calendars are more complex. They determine the month based on lunar phases, ensuring that each month starts on a fixed phase. However, to account for the mismatch between 354 days in a lunar year and 365 days in a solar year, these calendars incorporate an intercalary month every 2-3 years. This extra month helps reconcile the difference between lunar and solar cycles. 19 solar years are roughly equal to 235 synodic months and hence, in most calendars, this pattern of intercalary months repeats after 19 years. This is called the Metonic cycle.

8.1 Indian Lunisolar Calendar

In the Indian Lunisolar calendar, months end with the new moon day, and the next month begins on the day following the new moon. The month's name is determined by the lunar mansion (*Nakshatras*) coinciding with the full moon day. Each "tithi" corresponds to a 12-degree interval of lunar elongation. The *tithi* of any given day is determined by the elongation of the moon at sunrise. A *tithi* can last anywhere between 20 to 27 hours. We know that the sun moves faster(in the sky) in December as compared to its slow speed in June. Also, the orbit of the moon is elliptical so its motion is also not constant. This variation in the speed of the sun and the elliptical orbit of the moon leads to some *tithis* getting repeated and others getting missed in the calendar. Let us take an example to explain it.

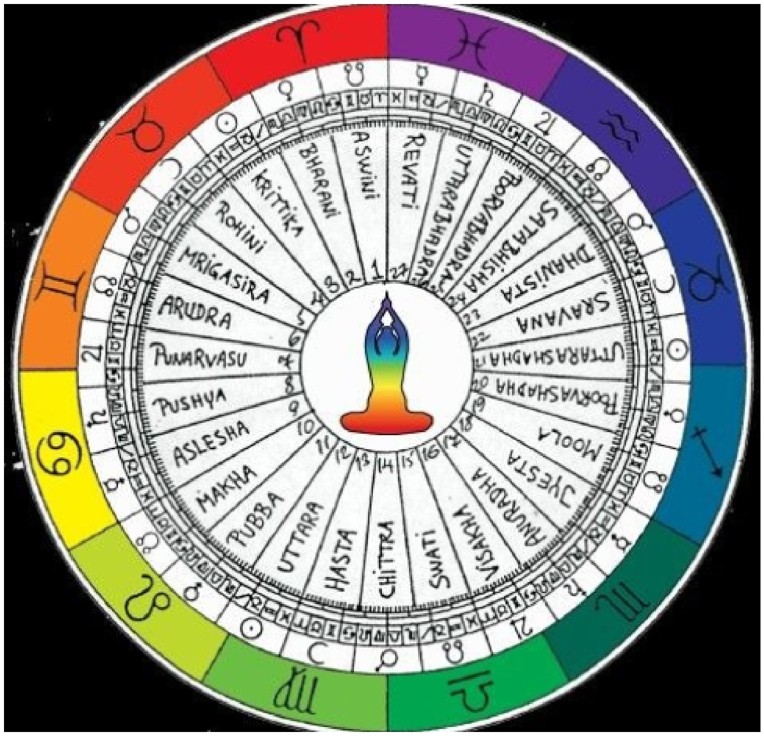

Figure 8.1: In Indian astronomy, the *Nakshatras* are divisions of 13 degrees 20 minutes starting from zero Aries and ending at 30 degrees of Pisces. The *Nakshatras* are referred to as the lunar mansions because the moon moves approximately $13°20'$ per day and, therefore, resides in one *Nakshatras* per day. A *Nakshatras* is one of 27 (sometimes also 28) sectors along the ecliptic.

Say *dwitiya* starts at 5:25 am and sunrise at local time is 5:30 am, whole day name is *dwitiya* until the next sunrise at 5:31 am (say). If *tritiya* starts at 5:29 (of the next day), there will be *tritiya* at sunrise, so the whole day is named as *tritiya.* If *tritiya* ends at 5:34 am of the third day and sunrise at 5:32 am. there will be *tritiya* in the next sunrise too. In this case, *tritiya* obtains two separate sunrises. The *tritiya* has repeated here (called *adhika tithi*). In the reverse case, sometimes a *tithi* starts after sunrise and ends before the next sunrise. In that case, there will not be any named lunar day (called *kshaya tithi*). Some *tithis* in the calendar may be missing or repeated, depending on the sunrise timings. This creates a unique characteristic of the Indian lunisolar calendar.

Since this is a lunisolar calendar, it uses intercalary months to keep in sync with the solar year. Intercalary months are taken after every 32 or 33 months, called *adhika masa* (extra month). The decision on whether a month is *adhika* is based on the movement of the sun. If the sun does not transit into a new zodiac sign (*rashi*) in a month, then it is considered *adhika.* It is named after the month that follows it. For example, if the *adhika masa* was before *Chaitra*, it will be called

Adhika Chaitra (extra Chaitra) and the next month will be *Nij Chaitra* (original Chaitra).

The most common surviving Indian lunisolar eras are:

- Shalivahan Shaka Samvat (starting from 78 CE)
- Vikram Samvat (starting from 56 BCE)

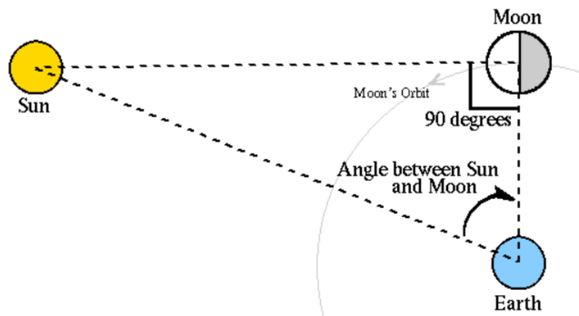

Figure 8.2: The angle between the sun and the moon shown above is what we refer to as the elongation of the moon

Meaning to some key words!

Elongation The angular distance in celestial longitude separating the moon or a planet from the sun

Dwitiya The second day of the waxing phase or waning phase of the moon. There are two of them in a lunar month with a time gap of a fortnight.

Tritiya The third day of the waxing phase or waning phase of the moon. There are two of them in a lunar month with a time gap of a fortnight.

8.2 Roman Calendar

8.2.1 The Old Roman Calendar

The Roman calendar was a lunisolar calendar with a unique structure. It consisted of 10 months totalling 304 days, from March to December. The remaining 50-odd winter days were either added to the last month or considered separately. The decision of when to start the next year was left to the local priest. While this system may seem peculiar, it worked for the people at that time, as they lacked education, and the winter season often confined them to their homes and it didn't matter for them which day of the month it was.

8.2.2 The Roman Republican Calendar

This calendar was a slightly improved version of the previous one. It consisted of 12 months totalling 355 days, with two months added at the beginning. This led to September, the seventh and October, the eighth month to be the ninth and tenth respectively, even though their names suggest otherwise. An intercalary month was added every 2 years. However, the decision to start the next year still rested with the local priest.

8.3 The Julian Calendar

When Julius Caesar came to power in Rome, he discovered that different provinces had varying opinions on when to start the new year due to abuse of power by local priests. This led to chaos when tax reports from different provinces did not align due to calendar discrepancies.

To resolve this issue, Caesar convened a conference of astronomers and sought their guidance. The astronomers recommended adopting a calendar where a year comprised of 365 days, with an additional leap day every fourth year, similar to what we follow today. This resulted in an effective year length of 365.25 days, close to the length of the tropical year.

Figure 8.3: Julius Caesar

The difference of 11 minutes 14 seconds per year accumulates to an error of 1 day in 128 years. However, since life expectancy during that period was around 40 years, this error was considered negligible over three generations. The new calendar started uniformly on 1 January 45 BCE. Then began a chaotic period of about 50 years, due to the death of Julius Caesar. During that period the priests accidentally declared a leap year every 3 years instead of 4, When Augustus Caesar became emperor, he realised that they had taken too many leap years and took a break for the next 12 years to make up for it. Finally, this system of one leap year every 4 years and 365 days a year settled down from 4 CE.

8.4 Gregorian Calendar

Was the calendar issue finally resolved and did everything run smoothly? Not quite! The 11-minute error reappeared in 1582 when it accumulated to 10 days, shifting the vernal equinox from 21 March to 11 March. In response, Pope Gregory called a meeting of astronomers to find a solution. The decision was made to skip 10 days and bring the calendar back on track. This was achieved by skipping days between 4 October and 15 October 1582. To prevent similar anomalies in the future, it was proposed that centuries should only be leap years if divisible by 400. This corrected the error by 3/400 days per year. The accuracy improved from 1 day in 128 years to 1 day in 3226 years. To illustrate this:
The errors written below are for one year
Initial error $= \frac{1}{128}$ day
Final error $= \frac{1}{128} - \frac{3}{400} = \frac{1}{3200}$ day
Since these errors are not exact, therefore we get 3200 instead of 3226, which is indeed the correct answer.

There was also a proposal to further refine the calendar by making years divisible by 4000 non-leap years, achieving an accuracy of 1 day in 20,000 years. However, this proposal is yet to be implemented, and future astronomers may handle it when the time comes.

8.5 Julian Dates

Astronomers devised a simpler way to track dates known as Julian dates, abbreviated as JD. Julian day 0 corresponds to noon on 1 January 4713 BCE in the Julian calendar. The day changes at noon in UT (Universal Time). Julian dates are calculated using straightforward subtraction to determine the number of days between two events, even if they are months or years apart.
Examples:
1 January 2000, 00:00 UT: JD 2,451,544.500
1 January 2020, 12:00 UT: JD 2,458,850.000

Futher Reading

1. Astronomy: Principles and Practice, 4^{th} Edition (PBK) by Archie Roy and David Clarke (Chapters 7 to 9)

2. Spherical Astronomy in Fundamental Astronomy by Hannu Karttunen

3. Textbook on Spherical Astronomy 6^{th} Edition by W.M. Smart