Distance Ladder

Learners' Space Astronomy

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What is Distance Ladder?

The cosmic distance ladder, also known as the extragalactic distance scale, refers to a sequence of methodologies employed by astronomers to determine the distances to celestial objects [\(Wiki\).](https://en.wikipedia.org/wiki/Cosmic_distance_ladder) It serves as a fundamental framework for comprehending the vastness of the universe and its components.

In a more concise and direct explanation, the concept of the distance ladder encompasses various techniques utilized in astronomy to measure distances. It serves as a comprehensive framework for understanding the spatial dimensions of celestial objects and phenomena.

1.1 The importance of Measuring Distances.

Before we delve into the topic of distance measurements in astronomy, let us explore why it is necessary to measure distances. While stargazing or observing celestial objects through telescopes, the immediate need for distance measurements may not be apparent to us. However, accurate distance measurements play a crucial role in advancing our knowledge of the cosmos. By understanding the significance of measuring distances, we can appreciate the value of dedicating time and resources to this aspect of study.

(a) A person looking through telescope (b) Celestial Sphere

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While it may seem that measuring distances is not essential for navigating stars in the celestial sphere, the significance of calculating distances to various objects in the universe should not be underestimated. By determining these distances, we gain invaluable insights into the actual threedimensional structure of the universe. This knowledge allows us to discern which objects are truly close and have a tangible impact on each other, as opposed to those that merely appear close from our perspective. Understanding the true spatial relationships between celestial entities contributes significantly to our overall comprehension of the universe. For example:

(a) This is how Orion constellation looks to us in the night sky.

(b) This is the actual relative distance between each stars

The importance of distances in astronomy can be highlighted through the following points:

- 1. Size: Calculating distances allows us to determine the true size of celestial objects. While some objects may appear larger than others, it does not necessarily mean that they are physically larger. The apparent size can be deceiving, as it may be influenced by the object's proximity. For instance, planets in the night sky may appear larger than other stars, but in reality, they are much smaller in size compared to most visible stars.
- 2. Energy: Distances to objects play a crucial role in determining their energy output. Luminosity, which measures the amount of energy produced by an object, can be affected by its distance from us. Farther objects tend to appear fainter, indicating a lower energy output. By knowing the distances to these objects, we can accurately determine the amount of energy they are producing.

These factors highlight the significance of measuring distances in astronomy, enabling us to unravel the true dimensions and energy characteristics of celestial objects.

The Need For Various Techniques in Distance Measurement

The need for employing different techniques to measure distances in astronomy is straightforward and arises from the significant variations in distances across the cosmos. The range of distances involved makes it impractical to rely on a single scale of measurement, as precision can diminish beyond certain ranges.

This concept can be easily understood through a simple analogy. Consider the task of measuring the distance between two walls within a room. For this purpose, a measuring tape suffices. However, when faced with the challenge of measuring the distance between distant cities like Mumbai and Delhi, utilizing a measuring tape becomes impractical and inefficient.

In such cases, alternative methods, such as satellite imaging, are employed. These methods are specifically designed to address the challenges posed by larger distances. Similarly, in astronomy, varying techniques are employed to overcome the limitations associated with different distance ranges.

In short, following are the main reasons why there are so many different ways to measure distances.

Figure 2.1: Distance Ladder

1. Varying Scales: The universe spans an immense range of scales, from nearby objects within our own galaxy to distant galaxies billions of light-years away. Each scale demands specific techniques tailored to its unique characteristics.

- 2. Distance Limits: Different techniques have varying ranges of applicability. For instance, parallax measurements are suitable for objects relatively close to Earth, while other methods, such as redshift and standard candles, are employed for more distant objects.
- 3. Object Characteristics: Celestial objects exhibit diverse properties, such as varying brightness, motion, and composition. Each object category necessitates specialized techniques that can account for these characteristics and derive accurate distance estimates.
- 4. Calibration and Validation: The reliability of distance measurements relies on cross-validation with other independent methods. Having multiple techniques allows astronomers to crosscheck and validate their findings, enhancing the overall confidence in the obtained distance values.

In this module, we will explore five methods used to measure distances on an extragalactic scale:

- 1. Parallax method: Utilizes the apparent shift in an object's position when observed from different vantage points to determine its distance.
- 2. Cepheid variables: Relies on the period-luminosity relationship of pulsating stars called Cepheids to estimate their distances.
- 3. Type 1a supernovae: Measures the absolute brightness of these specific supernovae, which serves as a standard candle for distance estimation.
- 4. Tully Fisher relation: Correlates the intrinsic luminosity of spiral galaxies with the rotational velocity of their gas or stars, providing a distance estimate.
- 5. Surface brightness relation: Establishes a connection between the observed surface brightness of an object and its distance, often applied to elliptical galaxies.

These five methods offer valuable tools in measuring distances on a cosmic scale, contributing to our understanding of the vast universe.

Chapter 3 Parallax Method

The method being described here is known as the parallax method, which is widely used in astronomy to measure distances. It relies on the principle of trigonometry and the baseline provided by the Earth's orbit around the Sun.

By observing the apparent shift in a star's position against the background of distant stars over a six-month period, astronomers can measure the parallax angle. This angle, combined with the known baseline of the Earth's orbit, allows for the calculation of the star's distance using trigonometric principles.

The parallax method is considered one of the most reliable techniques for measuring distances to nearby stars, providing valuable insights into the spatial dimensions of the universe.

Figure 3.1: Trignometric Parallax

As we can see in this image, if we take a particular star and observe its position against the distant background of stars at different times of year, we can actually work out what is that angle p.

Scientists actually measure how much a star has moved in radians from earth and that would come out to be the same as the angle earth has moved with respect to that star.

Now after knowing that angle it's fairly easy to calculate the distance between earth and the star given that we know the distance between earth and sun (this is called 1 AU).

There are reliable methods available to calibrate 1 Astronomical Unit (1 AU), which measures the average distance between Earth and the Sun. One such method involves bouncing radar off the surface of Venus at different time intervals. By carefully analyzing the round-trip travel time of the radar signals and employing geometric calculations, scientists can accurately determine the value of 1 AU.

[Here](http://spiff.rit.edu/classes/phys445/gettys/venus_ex/venus_ex.html) is the link for further reading on this topic.

Figure 3.2: Measuring distance with the help of radar bouncing off Venus

Here, we have to just measure the distance between Earth and Venus at their greatest elongation and when they are in a straight line with the sun. With the knowledge of the angle alpha, the distance between earth and sun could be calculated easily.

For your reference:

$$
1 rad = (180/\pi)^{\circ}
$$

\n
$$
1^{\circ} = 60'
$$

\n
$$
1' = 60''
$$
\n(3.1)

So, let's come back to measuring parallax, we have the value of angle 'p' very small, so that $\tan p \approx 'p'.$

Hence,

$$
D = 1AU/p \text{ radians}
$$

\n
$$
\rightarrow D = 1AU \cdot 206265/p \text{ arcseconds}
$$

\nWe take, 1 parsec = $1AU \cdot 206265$
\n
$$
\rightarrow D = 1/p(\text{in arcseconds}) \text{ parsecs}
$$
\n(3.2)

This means we can say that for 1 arcsecond of parallax angle we have the star at a distance of 1 parsec.

$$
1 parsec = 3.26
$$
 light years

Facts!

The ESA mission Hipparchos, launched in 1989, revolutionized parallax measurement and distance estimation in astronomy. It measured positions, parallaxes, and proper motions of 120,000 stars, providing valuable data for understanding the Milky Way.

However, in 2013, ESA launched Gaia, a telescope that surpassed Hipparchos with its ability to chart the positions, parallaxes, and proper motions of over one billion stars. This breakthrough allowed astronomers to create a dynamic movie of the galaxy's evolution, uncovering past events and projecting future changes. Gaia's advanced technology, including a billionpixel detector and larger mirrors, enables it to observe thousands of stars simultaneously and collect light more efficiently, providing deeper insights into the Milky Way.

Facts!

Did you know that when measuring the parallax of a celestial object, the path traced by the object's apparent position forms an ellipse? This phenomenon is known as the parallactic ellipse. The shape of the ellipse is determined by the object's distance from Earth and the baseline between observing positions. The longer the baseline or the closer the object, the more elongated the ellipse becomes. Interestingly, the major axis of the parallactic ellipse is perpendicular to the direction of the baseline. By studying the size and shape of the parallactic ellipse, astronomers can extract valuable information about the distance and motion of celestial objects, providing key insights into the vastness of the universe.

Chapter 4 Standard candles

Standard candles are celestial objects used in astronomy for distance determination. They possess well-defined absolute magnitudes that are assumed to remain constant irrespective of their age or distance. Examples of standard candles include Type I and II Cepheids, as well as RR Lyrae stars.

Cepheids are characterized by a period-luminosity relationship, meaning Cepheids with the same period have the same absolute magnitude. By measuring the apparent magnitude of a Cepheid and applying the period-luminosity relationship, astronomers can accurately determine its distance. If two Cepheids with the same period have different apparent magnitudes, the fainter one is assumed to be further away.

Figure 4.1: Standard Candles

Similarly, RR Lyrae stars can also be used as standard candles, although their intrinsic luminosity is lower than that of Classical Cepheids. Therefore, they are typically detectable at shorter distances.

Type Ia supernovae can also serve as approximate standard candles, as their absolute magnitude can reach around -19 at maximum brightness. Due to their extreme luminosity, they can be used to probe much greater distances in the universe than Cepheids. Research projects such as the Supernova Cosmology Project and the High-Z SN Search have observed numerous supernovae in distant galaxies to determine important cosmological parameters and unveil the accelerating expansion of the universe.

More about these standard candles will be discussed in the next few chapters.

Chapter 5 Cepheid variables

While the parallax method is very accurate for all practical purposes, due to limitation in the accuracy to measure angle, the maximum precision that could be reached is around 100 parsecs.

Here we will discuss another method to measure distance which will use the unique relation between luminosity and period of oscillation of Cepheid variables stars.

Cepheid variable stars are a type of pulsating star that exhibits regular variations in brightness over time. These stars are named after Delta Cephei, the first discovered star of this kind. It is a type of variable star that pulsates radially, varying in both diameter and temperature. It changes in brightness, with a well defined stable period and amplitude.

Figure 5.1: Cepheid variable

Cepheid variable stars were named after the first of their kind observed, δ Cepheus. There are actually two classes of Cepheid: **Type I Cepheids** (δ Cepheus is a classical Cepheid) are population I stars with high metallicities, and pulsation periods generally less than 10 days. Type II Cepheids (W Virginis stars), are low-metallicity, population II stars with pulsation periods between 10 and 100 days.

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Population I and Population II stars are distinct stellar populations in the Milky Way. Population I stars are young and metal-rich, found in the galactic disk, associated with star-forming regions, open clusters, and spiral arms. Population II stars are old and metal-poor, located in the galactic halo and globular clusters.

All Cepheids are luminous, yellow, horizontal branch stars that lie in the instability strip of the Hertzsprung-Russell diagram. Instabilities which cause their size and temperature to change

Figure 5.2: Cepheid variable in HR diagram

give rise to the periodic variations in their luminosity.

In 1907, Henrietta Leavitt discovered that Cepheid variable stars in the Small Magellanic Cloud pulsated at a rate which depended solely on their absolute magnitude. This period-luminosity relationship (shown below) allows Cepheids to be used as standard candles (once the pulsation period is known) to estimate distances to the objects in which they are located.

Both Cepheid variables and RR Lyrae exhibit distinct period luminosity graph shown below.

Figure 5.3: Period vs Luminosity

Now let's see how we can actually use the above given graph practically to measure distance. Let's say that we are dealing with a Type 1 Cepheid, but this method remains the same for Type 2 and RR Lyrae type stars.

Figure 5.4: Periodicity graph

- 1. Photometric observations, be they naked-eye estimates, photographic plates, or photoelectric CCD images provide the apparent magnitude values for the Cepheid.
- 2. Plotting apparent magnitude values from observations at different times results in a light curve such as that below for a Cepheid in the (Large Magellanic Clouds) LMC.
- 3. From the light curve and the photometric data, two values can be determined; the average apparent magnitude, m, of the star and its period in days. In the example above the Cepheid has a mean apparent magnitude of 15.56 and a period of 4.76 days.
- 4. Knowing the period of the Cepheid we can now determine its mean absolute magnitude, M, by interpolating on the period-luminosity plot. The one shown below is based on Cepheids within the Milky Way. The vertical axis shows absolute magnitude whilst period is displayed as a log value on the horizontal axes.

Figure 5.5: Absolute magnitude vs log of periodicity

After getting the apparant magnitude and absolute magnitude, we can use the following formula to relate distance to these measurements.

$$
m - M = 5\log d - 5
$$

After rearranging this we can get:

 $d = 10^{(m-M+5)/5}$ parsecs

Where,

 $M =$ absolute magnitude $m =$ apparent magnitude $d =$ distance to the star

This method allows us to measure distances from 1 kpc to 50 Mpc

After substituting the values given in the example distance should come out to be **68230 parsecs**.

Type Ia supernovae

Type Ia supernovae is a type of supernovae that occur in binary star systems, where one star is a white dwarf and the other can be a giant star or another white dwarf.

White dwarfs with a mass below the critical limit of about 1.44 solar masses undergo carbon fusion and explode as supernovae. Chandrasekhar mass is the threshold for this explosion.The Chandrasekhar limit is the maximum mass of a stable white dwarf star. Currently accepted value of this is about 1.4 $M_{\odot}(2.765 \cdot 10^{30} kg)$.

Originally, astronomers classified supernovae into two types based on their observational characteristics: Type I and Type II. Type I supernovae lacked Hydrogen emission lines in their spectra, while Type II supernovae exhibited Hydrogen emission lines. However, further research revealed the existence of three distinct subtypes within Type I supernovae: Type Ia, Type Ib, and Type Ic.

Type Ia supernovae (SNIa) are believed to result from the explosion of a carbon-oxygen white dwarf in a binary system that surpasses the Chandrasekhar limit. This can occur through either accretion from a donor star or through mergers. SNIa are the brightest type of supernovae, with an absolute magnitude of approximately -19.5 at maximum light. They are found in all galaxy types and are characterized by a silicon absorption feature in their spectra at a rest wavelength of 6355 Å. These supernovae can expel material at speeds around $10,000 \text{ km/s}$ and can outshine an entire galaxy at their peak brightness.

Figure 6.1: G299 Type 1a Supernovae remnant

The gravitational pull of the white dwarf causes it to take matter from its companion star. Eventually it reaches a high enough mass(about 1.44 solar masses) that it cannot support itself against gravitational collapse and explodes.Below is an image which explains this process in a systemic way. [Here](https://youtu.be/5YZkAoR3WLE) is a cool youtube video link for type Ia supernova.

Figure 6.2: Process for Type 1a supernovae

As mentioned earlier, all Type Ia supernovae reach nearly the same brightness at the peak of their outburst with an absolute magnitude of -19.3 ± 0.03 . Now we can measure the apparent mag-

Figure 6.3: Luminosity vs Time graph

nitude from earth and again use the below given relation to relate absolute brightness, apparent brightness and distance from that supernovae.

$$
m - M = 5\log d - 5
$$

This method is used to measure distances from about 1 Mpc to over 1000 Mpc.

A discernible pattern emerges when considering both of these methods: Type Ia supernovae and Cepheid variables. These methods rely on our ability to determine the intrinsic brightness, also known as the absolute brightness, of these celestial objects. Once we have this intrinsic brightness

$$

information, our task becomes considerably simpler.

From Earth's vantage point, we can measure the apparent magnitude of these objects and utilize the aforementioned relationships to calculate their distances. This straightforward approach allows us to leverage the intrinsic brightness of Type Ia supernovae and Cepheid variables, providing us with an effective means of determining astronomical distances.

These objects are commonly referred to as "Standard Candles" in astronomy.

Tully Fisher relation

The Tully-Fisher relation, named after astronomers R. Brent Tully and J. Richard Fisher, emerged as a significant breakthrough in the field of distance measurement in the late 1970s. It established a correlation between the intrinsic luminosity (brightness) of spiral galaxies and the rotational velocity of their gas or stars. This relationship allowed astronomers to estimate distances to spiral galaxies based on their observed rotational velocities.

Prior to the Tully-Fisher relation, distance measurements to galaxies relied on indirect methods or were limited to nearby objects. The discovery of this empirical relationship opened up new possibilities for estimating distances to galaxies on a much larger scale.

It is an empirical relationship between mass or intrinsic luminosity of a spiral galaxy and its asymptotic rotation velocity or emission line.

It is used to derive galaxy distances in the range of 7 to 100 Mpc.

Luminosity is proportional to some power of the mass. When the luminosity of main sequence stars is plotted against their masses, we observe a mass-luminosity relationship, approximately of the form $L \propto (M)^{3.5}$.

Rotational velocities are measured from hydrogen line spectral observations. The rotation of the galaxy causes the hydrogen line to be doppler shifted.

The equation is given below:

$$
\nu_{obs} = \nu_{rest} \cdot \sqrt{\frac{(1 - v/c)}{(1 + v/c)}}
$$

where ν_{obs} is the observed frequency of the line emission from the gas, ν_{rest} is the frequency of the line emission when the gas is not moving, v is the velocity of the gas, and c is the speed of light.

The Tully Fisher relation is the most fundamental property of spiral galaxies. Specifically it is found that for luminosity and rotation velocity, we have the below relation:

$$
L \propto (v_{rot})^{\alpha}
$$

One important point is that α here does not have a unique value. The details of both the photometric and spectroscopic measurements affect it. A typical result found in contemporary studies put α very close to 3. So, how do we use all this information to actually calculate distances to galaxies?

We take data for the flux of the galaxies from the telescopes. Telescopes actually measure the flux, then it could be converted to luminosity with the help of the following relation:

$$
f=\tfrac{L}{4\pi D^2}
$$

Remember, 'D' is the quantity that we need to calculate here.

So for galaxies with known distances (through other methods like Cepheid variables or Type 1a supernovae) , we find this value of luminosity (L). As mentioned earlier, we can relate hydrogen line spectrum with rotational velocity of the galaxy. Or we can deploy some other methods to calculate V_{rot} for the galaxy.

Then we have to plot the graph between the log of luminosity and log of rotational velocity. After

Figure 7.1: Luminosity vs Rotational Velocity

using a best fit line on the graph we can now calculate distance.

For galaxies with unknown distances, we can use the best fitted line and data of V_{rot} to find the value of luminosity. We can then relate this luminosity with observed flux of the galaxy with below formula:

$$
f = \tfrac{L}{4\pi D^2}
$$

If you want to know more about this method, check these websites below, they serve as excellent resources for this method.

[A real example of measuring distance using Tully Fisher relation](https://www.alma.ac.uk/nas/gbendo/Web/Sci/Teach/experiment_tullyfisher.pdf) [Tully Fisher relation for spiral galaxy.](https://ned.ipac.caltech.edu/level5/Willick/Willick3.html)

Surface Brightness Fluctuations

SBF is a technique used to measure distances to galaxies with high precision. It is based on the observations that closer galaxies exhibit more visible fluctuations in brightness due to distribution of stars within them. By analyzing the pixel to pixel variations in the surface brightness of galaxies, SBF allows us to calculate distance to the galaxies.

This method plays a crucial role in understanding the large scale structure of the universe. It is used mainly in the 10 to 100 Mpc range.

Among the available extragalactic indicators, the surface brightness fluctuations method is one of the most accurate and precise, permitting measurements of individual distances at $\langle 5\%$ precision.

Some of the recent discoveries where this method was used are, distance to Messier 87 (where first "image" of black hole was taken) , distance to NGC1052-DF2 and distance to NGC4993. Let's understand what these surface brightness fluctuations actually are.

(a) Nearby galaxy having more details, hence "grainy" image

(b) Far away galaxy, having more "smooth" structure

As you can see in the above image, on the left hand side we have an elliptical galaxy which is much closer as compared to the one on the right hand side. We can clearly see in the above image that the closer galaxy exhibits more structure and looks granular in nature, while the farther galaxy seems much more "smooth".

So this means that this granular and smooth structure is somehow related to the distance of the galaxy.

Figure 8.2: Reference image 1

Figure 8.3: Reference image 2

Consider the above given image, on the left hand side we have a galaxy at a particular distance and on the right hand side we have a galaxy which is 3 times further away than the previous ones. Recall that the flux is inversely proportional to the square of distance and stellar density would be directly proportional to the distance square.

We define surface brightness as a product of this flux and number density.

A link to interesting article related to this is given [here.](https://en.wikipedia.org/wiki/Olbers%27_paradox#:~:text=Olbers)

As we can see from the above diagram that surface brightness would remain constant in every pixel of ccd.

Now if we increase the number of pixels, we can see that in the image on the left side there are more fluctuations pixel to pixel.

These fluctuations are characterized by the square root of n multiplied by flux.

In the "reference image 2" we are dividing rms squared with mean to get a normalized value for flux which would account for any atmospheric disturbances.

$$
\overline{L} = \frac{\sum_{j} n_j L_j^2}{\sum_{j} n_j L_j}
$$

Above we are calculating mean luminosity which is defined to be the ratio between 2nd moment to the 1st moment of luminosity function.

The first moment represents the average brightness, while the second moment characterizes the distribution or scatter of luminosities.// within a population of celestial objects. Above n corresponds to the objects having luminosity between L_{j-1} to L_j .

After calculating the above thing we can now easily find absolute magnitude according to the following relation.

$$
\overline{M} = -2.5 \log \frac{\sum_{j} n_j L_j^2}{\sum_{j} n_j L_j} + z.p
$$

z.p refers to some zero point calibration.

The zero point is used to calibrate a system to the standard magnitude system, as the flux detected from stars will vary from detector to detector.

The zero point of an instrument, by definition, is the magnitude of an object that produces one count (or data number, DN) per second. The magnitude of an arbitrary object producing DN counts in an observation of length EXPTIME is therefore:

$$
m = -2.5 \cdot \log \left(\frac{DN}{EXPTIME} \right) + ZEROPOINT
$$

Let's now briefly look at how we actually calculate the distance after learning the above things.

- Firstly we remove the bright objects from the galaxy's image to reveal the underlying stellar population through various methods.
	- One thing to remember here is that we do not want to remove bright stars from our images because they are the ones which actually give pixel to pixel fluctuations. We only want to remove bright globular clusters, dust and other galaxies in between.
- Now the next part is slightly more technical, feel free to explore other resources if you think you want to know more about these techniques.
	- After getting a clean image, it is Fourier transformed to get 1D azimuthal-averaged power spectrum.
- The power spectrum is then fitted with a combination of the power spectra of the PSF model convolved with those of the mask and a constant that represents photometric noise. The PSF is modeled using the image of a nearby star.
- From the residuals, the apparent magnitude of the fluctuations is calculated. Combining this with the absolute magnitude determined through an independent calibration of the underlying stellar population, we can then find the distance modulus.

Conclusion and further reading

In conclusion, the comprehensive examination of five distinct methods- parallax, Cepheid variables, Type 1a supernovae, Tully-Fisher relation, and surface brightness fluctuations– has significantly enhanced our capability to determine distances in the vast realm of extragalactic scales. Each method's unique strengths and limitations contribute to a more refined understanding of cosmic structures and their evolution.

Below are resources you can check out for more in-depth knowledge of these topics(other than wikipedia, which you can refer anytime). They have been used extensively in creating this module also. [Star distances and Trigonometric Parallax.](https://www.astronomynotes.com/starprop/s2.htm) [Type Ia Supernova](https://astronomy.swin.edu.au/cosmos/t/Type+Ia+Supernova) [Cepheid Variable Stars, Supernovae and Distance Measurements](https://lco.global/spacebook/distance/cepheid-variable-stars-supernovae-and-distance-measurement/#:~:text=Using%20Cepheid%20Variables%20to%20Measure%20Distance&text=Once%20the%20period%20of%20a,its%20distance%20can%20be%20determined) [An Example of Distance Determination Using Cepheid Variables](https://www.atnf.csiro.au/outreach/education/senior/astrophysics/variable_cepheids.html#:~:text=All%20Cepheids%20with%20a%20certain,it%20must%20be%20further%20away) [About Cepheid Variables](https://astronomy.swin.edu.au/cosmos/c/cepheid+variable+stars#:~:text=There%20are%20actually%20two%20classes,between%2010%20and%20100%20days) [Deivation of Distances using Tully-Fisher Relation](https://arxiv.org/abs/0806.1140#:~:text=The%20Tully%2DFisher%20relation%20is,interval%207%20to%20100%20Mpc) [Extragalactic distances with Surface brightness fluctuations- A very nice Talk](https://www.youtube.com/live/fSJsXsT7xJk?feature=share) [Another Wonderful Talk on Surface Brightness Fluctuations](https://youtu.be/LQcBKiKiMbw) [Surface Brightness Fluctuations;History and Intro](https://ned.ipac.caltech.edu/level5/Jacoby/Jacoby9_1.html) [Tully Fisher relation for Spiral Galaxies](https://ned.ipac.caltech.edu/level5/Willick/Willick3.html) [A Real example of Measuring Distances to Galaxies using the Tully-Fisher Relation](https://www.alma.ac.uk/nas/gbendo/Web/Sci/Teach/experiment_tullyfisher.pdf)