KRITTIKA SUMMER PROJECTS 2024

BINARY BLACK HOLES FROM SCRATCH

WITH COMPAS

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Abstract

Gravitational waves, ripples in spacetime initially predicted by Albert Einstein in 1916 through his General Theory of Relativity, were first experimentally detected in 2015 by the Laser Interferometer Gravitational-wave Observatory (LIGO). To model the gravitational wave signals observed by LIGO, it is necessary to solve the Einstein Field Equations to theoretically generate the waveforms. Given the equations' highly non-linear and complex nature, approximation methods are employed. This project aimed to generate gravitational waveforms for Compact Binary Coalescence (CBC) using the Quadrupole Approximation and Post-Newtonian Expansions across various source parameters. The Compact Object Mergers: Population Astrophysics with Stellar Simulations (COMPAS) framework is a sophisticated tool designed to study the evolution of binary stellar systems and the formation of compact objects such as black holes, neutron stars, and white dwarfs. By incorporating detailed physical models for processes like mass transfer, supernova kicks, and common envelope evolution, COMPAS provides insights into the rates, distributions, and characteristics of compact object mergers. These simulations are crucial for interpreting gravitational wave observations and enhancing our understanding of stellar evolution and binary interactions. Binary black hole (BBH) systems are key targets in the study of gravitational waves and the evolution of massive stars. COMPAS is the perfect and powerful framework tool for simulating the formation and evolution of BBH systems. These simulations provide critical insights into the physical processes driving the formation of BBH systems and help interpret gravitational wave detections by observatories like LIGO.

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Stellar Evolution

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A fundamental concept in astronomy is the notion that stars undergo significant changes over time. Stars are born from clouds of interstellar gas and dust, generate energy through the nuclear fusion of hydrogen in their cores, and eventually exhaust their fuel, dying and returning some of their mass to interstellar space. These remnants can then be incorporated into new generations of stars, perpetuating the cycle. This entire process is known as stellar evolution. However, stellar evolution occurs over millions or even billions of years, far exceeding the timespan we can observe directly. So, how do we know that stellar evolution takes place despite these lengthy timescales?

Variable stars reveal a crucial aspect of our universe: it is always in flux. Despite the vastness of the universe, the immense distances between stars and galaxies, and the long timescales over which many changes occur, most celestial objects—like stars, nebulae, and galaxies—seem unchanging within a human lifetime. However, variable stars defy this apparent stasis, exhibiting changes over periods we can observe.

These stars have been found to vary on timescales ranging from milliseconds to centuries. Each variable star offers insights into its own nature through its variability. The information gleaned from studying variable stars has significantly enhanced our understanding of the cosmos and its dynamic nature.

When astronomers observe stars, they gather valuable data on how these stars behave. By building and testing hypotheses on why stars vary, and refining these hypotheses with the accumulated data, astronomers can develop increasingly accurate descriptions of stellar variability. Each piece of evidence offers a unique test, contributing to a more comprehensive understanding of stellar behavior.
By studying individual variable stars in detail, we can extend our knowledge to broader classes of stars. Eventually, this knowledge allows us to understand all stars, by integrating models and descriptions of different star types. This approach enhances our understanding of what stars are and how they evolve.

1.1 The Hertzsprung-Russell diagram

There are various physical characteristics of stars that provide important information on the lives of stars. Two quantities, **mass** and **age**, are probably most fundamental. The progress of a star's life is predestined by its mass, because ultimately the mass determines how much energy the star can produce and how quickly it will do so. However, with mere observation, it is very hard to measure directly these two quantities. Two other parameters are a star's luminosity and temperature. Both of these are related to mass and age in a way that we now understand. Now it seems ideal to find a way to classify stars based upon simple observation of their parameters.

In the early 20th century, astronomers Ejnar Hertzsprung and Henry Norris Russell made a groundbreaking discovery for comparing different stars. They observed that when plotting the brightness of stars against their spectral type or color, the stars fell into well-defined regions on the graph. This relationship indicated that a star's brightness corresponded to a specific range of colors, and vice versa. Further research revealed that the resulting color-magnitude diagram, or Hertzsprung-Russell (H-R) diagram, provides a snapshot of the evolutionary states of the stars it represents. Stars occupy different regions of the H-R diagram based on their masses and ages. As stars age, they change in brightness and color in predictable patterns, with stars of varying masses following distinct evolutionary paths.

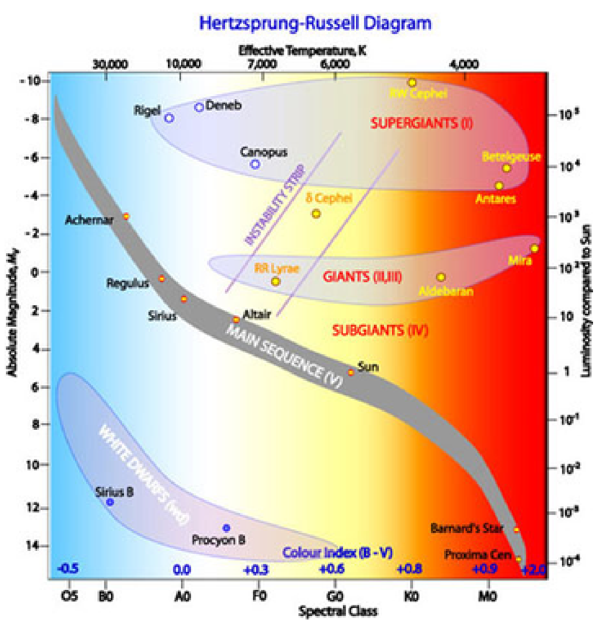


Figure 1.1 The H-R diagram showing evolutionary path

Why is this concept crucial for understanding variable stars? Each star possesses unique physical properties and occupies a specific position on the H-R diagram. If a star exhibits variability, analyzing this variability can reveal valuable physical information about stars located in similar positions on the H-R diagram. Given that variable stars span different regions of the H-R diagram, studying them has provided significant insights into stellar evolution. This approach allows us to learn about the life cycles of stars, even though the evolutionary processes for any individual star might span millions or billions of years. By examining variable stars, we can infer characteristics and behaviors of various star types, thereby understanding stellar evolution.

1.2 Working principle of a star

Throughout a star's lifespan, as observed in the H-R diagram, it goes through various phases, each with its unique way of generating energy in the form of light. Given that the universe is primarily composed of hydrogen and helium, it’s natural to wonder how the wide variety of elements, including unstable ones that decay into simpler elements through radioactive decay, are created. The core of a star acts as a factory for these elements, producing them during different phases of stellar evolution. Each stage in a star’s life contributes to the synthesis of new elements, enriching the universe with the diverse array of elements we observe today.

We’ll discuss various phases of a star soon, but just giving some glimpse of it, generally star follows the following track:

Nebula of dust and gases -> Main sequence star -> Red Giant/ Super Red Giant -> Dwarf Star -> Neutron star/Black hole (If it exceeds certain limit)

In between each phase, star shows some sort of variability, which indeed helps us to know about various parameters of stars. Starting with the mechanism protostar have to produce energy.

A protostar forms from a dense region within a molecular cloud. As the gas and dust compress, the density and pressure increase in the core of the collapsing region. As the material falls inward, gravitational potential energy is converted into thermal energy. This causes the temperature of the protostar to rise. The increasing temperature in the core of the protostar leads to the emission of thermal radiation, which is initially in form of infrared radiation, and further increment of mass of dense region causes emission of visible light.

A main sequence star produces light through the process of nuclear fusion, primarily converting hydrogen into helium in its core. In the core, the temperatures are extremely high (around 10 to 15 million Kelvin) and the pressures are immense. These conditions are sufficient for nuclear fusion to occur. Figure 1.2 shows the sequential nuclear reaction.

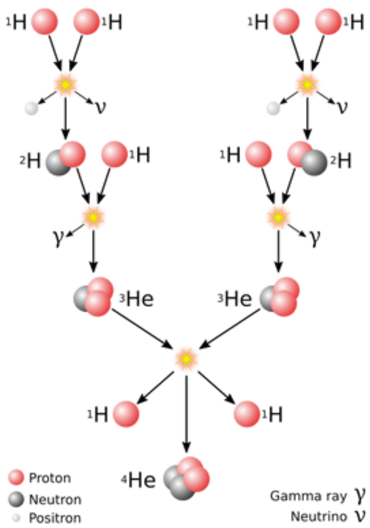
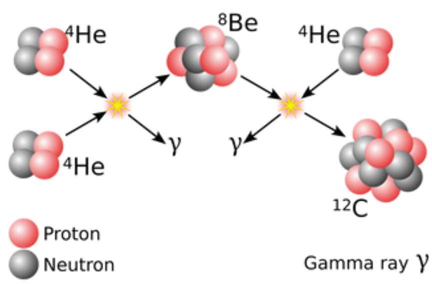


Figure 1.2 The chain of nuclear reaction inside the burning core of a main sequence star

As all the hydrogen in the core get depleted, star tends to adjust itself, as it was the thermal pressure which was keeping it's size intact. As the star starts shrinking, its Hydrogen from outer layer undergoes nuclear fusion, thus expanding its outer shells. The moment core is hot enough, it will start burning it's Helium and thus producing energy by the process of Triple Alpha, in it's Red Giant phase. Figure 1.3 shows the sequential reaction.

Figure 1.3 The chain of nuclear reaction inside the burning core of a Red or Super Red Giant



White dwarfs are luminous due to their residual thermal energy. Due to degeneracy pressure, if white dwarf exceeds certain limit, they either become neutron star or a black hole. Neutron star too emit light by residual thermal energy, or by pulsar emissions, where they emit beams of electromagnetic radiation through their magnetic poles. Black holes as the name suggest are invisible, as their gravitational pulls are so strong that light can't escape from certain point, called as event horizon. Though, the material in accretion disk heats up and emit light, apart from the theoretical 'Hawking radiation'.

1.3 Phases of a star

1.3.1 Protostar

Every star that you see in the sky was once formed inside a star forming region, millions or billions of years ago. Concentrations of dust and gas, known as 'Nebulas' are such regions. After the gravitational collapse and accretion of gases, a protostar is formed, which itself goes through a variability phase. The most famous class of these nebular variables are the T Tauri stars, named for the prototype, T Tauri. Following is image from JWST's NIRCam of L1527, in constellation of Taurus.

Figure 1.3.1 A molecular cloud that harbors a protostar.

In this Pre-Main Sequence phase, the root cause of variability of star is rapid accretion of circumstellar material onto the young protostar for a period of a few years, making them brighter than usual for certain period of time. The other way, where the protostar gets dimmer than usual is due to clumsy circumstellar disk. Some of these clumps are large enough to partially obscure the protostar as they orbit around it, causing the star to dim before our eyes.

1.3.2 The main sequence

After the protostar has become massive enough, and the core is enough hot, it starts to burn Hydrogen in its core and shines as an actual star. The main sequence is defined as the part of a star's lifetime spent burning hydrogen at its core; the start of its main-sequence lifetime is the point at which hydrogen burning first begins, and the end is defined by the point at which it runs out of hydrogen in its core. Less massive stars have larger lifespan, as their rate of burning Hydrogen is less than that of a more massive star.

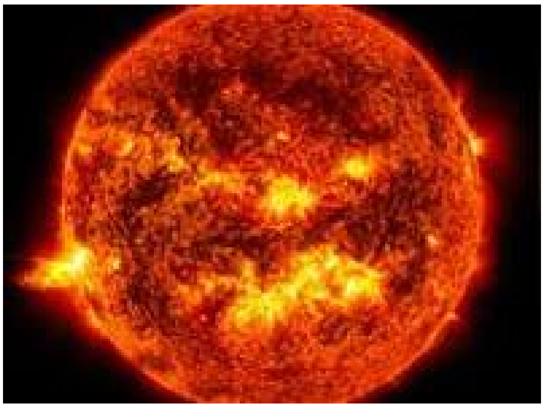


Figure 1.3.2 Sun in it’s Main Sequence phase

The Sun is perhaps the most important pulsating variable there is, and the study of its pulsations is called helioseismology. Studies have revealed that there are thousands of pulsation modes present inside the Sun at any given time. The way geologists study interior of Earth by observing the vibrations, helioseismologists do the exact same. Another kind of variability Main Sequence stars show on surface are by dark spots. When strong magnetic field interfere with convection region of a star and thus interfering with movement of gas, energy can’t get out easily. These is the reason why we see ‘sunspots’ on the surface of the Sun.

Now lets dive into the post main sequence phase. The instability strip runs from upper right (luminous and cool) to lower left (faint and hot) in the H-R diagram. Star lying in this strip may begin to pulsate. In stars, specific layers can become more opaque to radiation as they heat up or cool down. When this occurs, energy from the star's interior can become trapped in that layer, leading to an increase in temperature and pressure. If this layer is situated at an optimal depth within the star, it can function like a piston, causing the outer layers to move up and down in a regular, periodic manner, resulting in the star's pulsation. In the instability strip, there are the Cepheid variables, named after the class prototype delta Cephei.

One crucial thing about the Cepheid variables is that the period of its pulsation cycle is proportional to its absolute luminosity, or brightness. This is known as the period-luminosity or P-L relation, and also by the name Leavitt Law. The formulation for the same is as follow. P is in days.

$$M_{\mathrm{v}} = (-2.43 \pm 0.12)\left(\log_{10} P - 1\right) - (4.05 \pm 0.02)$$

With the absolute luminosity known, and the observed apparent luminosity, we can find distance of a Cepheid variable and hence, finding distance of any galaxy holding the Cepheid. The relation of observed and absolute luminosity, and distance is as follow. Dl is in parsecs.

$$M = m - 5\log_{10}\frac{D_L}{10\,\mathrm{pc}}$$

1.3.3 The cooler yet brighter Giants

Over the time when Hydrogen in the core is depleted and left with inert Helium core, the core stars to contract under its own gravity due to no thermal pressure. As core contracts, the temperature and pressure rises, due to which, the shell containing Hydrogen starts igniting, producing a lot more energy than that of burning Hydrogen core. Such immense pressure causes outer layer of star to expand, and consequently, those layer cools down. Despite of being cool, they are more luminous due to their larger size. This is how, a Red Giant or maybe a Super Red Giant depending on size, is formed. Due to development of a ‘convection envelope’, the Helium and other heavier elements starts falling onto the inert core. After the temperature of core reaches certain point, the inert Helium core gets active, and nuclear fusion of Helium starts, producing Carbon and Oxygen via Triple Alpha process.

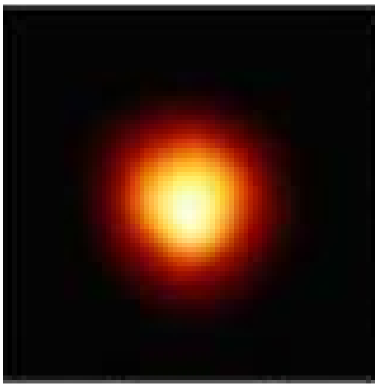


Figure 1.3.3 Direct image of Betelgeuse, a Red Giant from Hubble telescope

1.3.4 Asymptotic Giant Branch

Asymptotic Giant Branch (AGB) stars represent the final phase of stellar evolution where a star actively shines due to thermonuclear reactions occurring within its core. After evolving through the Red Giant Branch (RGB) phase, these stars possess a core predominantly composed of carbon and oxygen, encircled by layers of helium and hydrogen. The burning Helium from the Helium shell falls off into the Carbon core and similarly, burning Hydrogen falls into Helium shell. These burning shells are the reason why the star puffs up its outer layers. Again, due to enormous size, they are cooler, yet they are very hot due to even more energetic thermonuclear processes going on deep inside.

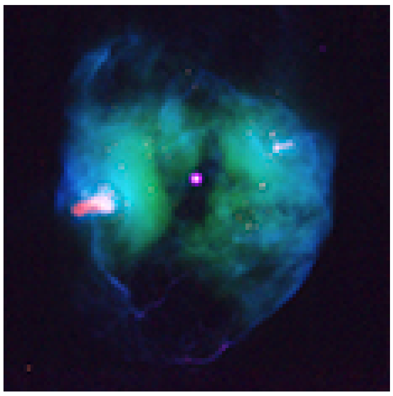
The AGB is the locus of one of the most famous and earliest-known classes of variable star, and one near and dear to variable star observers: the Mira variables. Just like Cepheid and other pulsators, these also follows the Period-Luminosity law. Though, these have very high rate of mass loss, as after these, they head towards being planetary nebula and white dwarf.

1.3.5 The Dwarfs

I the main sequence stars, gas inside a star obeys physical rules of equation of state. But as the star age, and the core becomes dense due to heavier elements, something interesting happens. Density increases to such a level that matter stops acting normally, and material becomes degenerate. meaning that the electronic fields of individual atoms can no longer keep them separated as they normally do. At this point, behavior of gas changes and it follows degenerate equation of state. Gas don’t respond quickly to heating or increasing temperature, as an ideal gas might. This stops the thermonuclear burning.

A star with a core in this condition is on the brink of its cosmic end. This dense, compact, and extremely hot core will become a white dwarf. As the star progresses, it will start shedding its outer layers, gradually revealing the white dwarf at its center. Eventually, only this white dwarf core will remain, marking the final stage of the star's life. As the material flows away into the space, it becomes more diffuse in nature, some of them lit by the stellar remnant within, forming what one see is planetary nebula.

Figure 1.3.4 Image of a planetary nebula with the remaining core of dead star, or a white dwarf



As the dead core cools down, it starts turning red, and then it’s termed as a Red Dwarf. Further cooling of Red Dwarf causes the dead core to loss it’s thermal luminosity, and turning into a theoretical black dwarf, which no longer emit significant heat or light.

1.3.6 More extreme destinies of star

If a star reaches the end of AGB and have a mass less then 1.4 Solar Masses, it will end it's life as a white dwarf. Whereas if not so, it will implode as it's own gravity is more stronger than the outward degeneracy pressure which was holding it against its gravity. The implosion is called a 'Supernovae', and the mass limit of 1.4 solar masses is called the 'Chandrasekhar Limit'. Degeneracy pressure is a quantum mechanical phenomenon that arises from the Pauli exclusion principle, which states that no two fermions can occupy the same quantum state simultaneously. As the core contracts under gravity, electrons are squeezed into a smaller volume. Due to the Pauli exclusion principle, they cannot all occupy the lowest energy state, so they fill higher energy states. This creates a pressure that does not depend on temperature but rather on the density of electrons. This pressure counteracts gravity and prevents further collapse, supporting the white dwarf.

The supernovae is one of the most energetic event in universe, unleashing runaway nuclear reactions that create every element in the periodic table along with a storm of subatomic particles that blast away the outer layers of the star at close to the speed of light. For a few months, the amount of energy released by a supernova can equal the combined light of every other star in a galaxy -- the light of a hundred billion stars or more.

From what's leftover is also dependent on the mass of the remanent. If the collapsed core is less than about 3 Solar Masses, the result will be an ultra dense object having diameter of about just 10 Kilometers! Their are variable neutron stars too. Due to shrinking a lot in size, they conserve angular momentum and thus they spin at an extreme rate. Pulsars are the one radiating energy from their magnetic pole while they are spinning at very high rate along with a rotating magnetic pole. An even more extreme variable neutron star is a magnetar -- a neutron star with a powerful magnetic field that undergoes enormous outbursts at high energies. Magnetars can emit huge amounts of high energy radiation detectable from across the entire Milky Way.

Even these scenarios do not represent the most extreme fate for massive stars. For stars exceeding roughly three solar masses, even the atomic forces that keep nuclei apart cannot prevent collapse under the star's immense gravity. This process leads to the formation of one of the universe's most enigmatic objects: a black hole. Black holes possess gravitational fields so intense that their escape velocities surpass the speed of light. Any matter or radiation that ventures within a certain distance — known as the event horizon — becomes irretrievably trapped, as nothing can exceed the speed of light to break free.

II

Binary Evolution

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Most stars in the universe are not alone; they are often formed in bulk. In fact, about half of all stars exist in either binary or multi-star systems. The presence of other stars significantly impacts their evolution. Here, we will explore how a star evolves within a binary system. Now that we've looked at how an isolated star evolves over time, let's dive into the fascinating dynamics of binary star systems.

2.1 Binary Stars

A binary star system consists two stars interacting with each other due to the fact that they are bound with the gravitational field. They tend to rotate about the common center of mass. These type of systems are contribute immensely to astronomers, as they can determine mass and other properties of star by taking some simple assumptions of conserving some physical quantity such as angular momentum. If calculated the time period of orbit, by using Kepler's law and Gravitational law, one can find the basic required properties of stars.

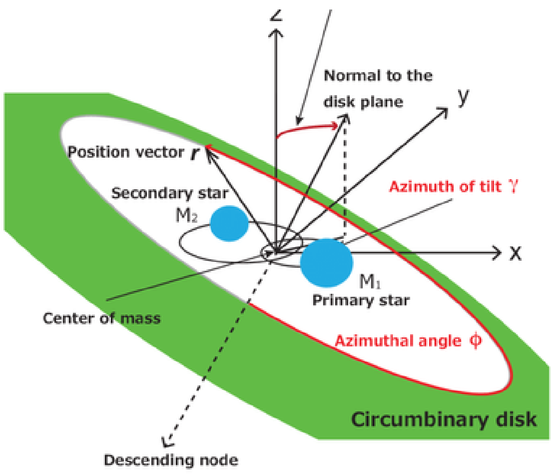


Figure 2.1 Image of a schematics of a binary star system

2.2 Binary Interactions

Stars in a binary star system interact primarily by mass transfer. The rate of mass transfer, rate of mass loss due to stellar winds, and how quick does a companion star evolves causes several types of final destinies of the system.

Mainly, the binary system evolution is dependent on the factors such as: Masses of star, Mass ratio of stars, separation between stars, eccentricity of orbit, stellar winds which causes non conservative mass loss and metallicity.

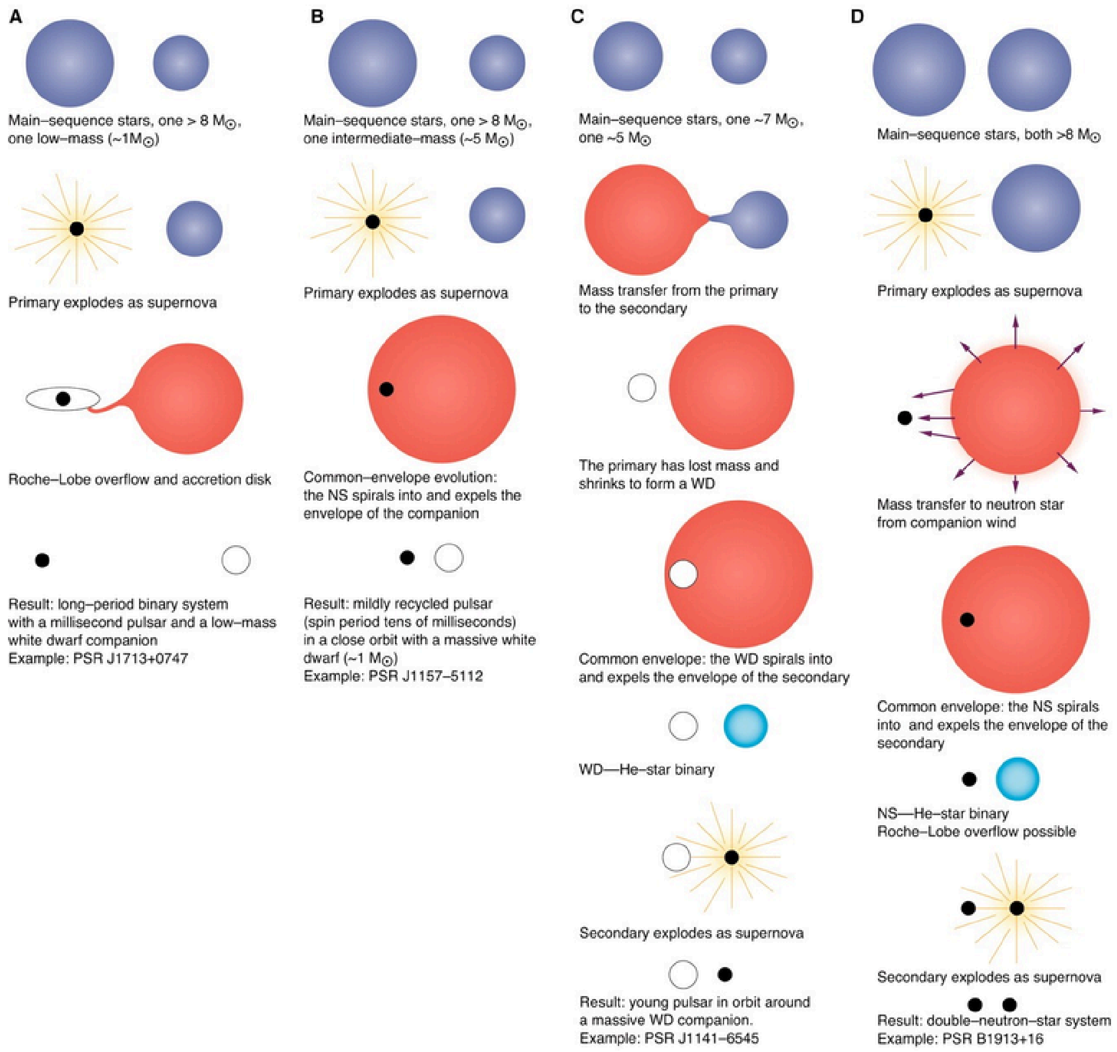


Figure 2.2 Different types of binary interactions

2.3 Mass Transfer

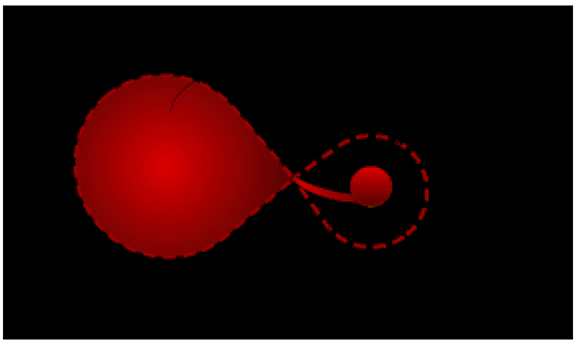
Star in binary system will either interact or just continue revolving around each other depending on factors of their initial separation and masses of both stars. Mass transfer is one of the most important event in this systems, which can change the way a star can evolve being alone.

We will discuss two main kind of mass transfer mechanism in a binary star system, that is - Stable and unstable mass transfer.

2.3.1 Stable Mass Transfer

Visualizing strength of gravitational force as a function of Gravitational potential around the binary star system will show some areas of negative force and some areas with positive force (Taking one of the star as reference point). If there exists a surface such that effect of gravity of both star gets nullified, such surface is called as 'Roche Lobe'. Such equipotential surface as every point on it which experiencing no gravitational force from any of the star.

Now, as star evolves over its life span, it grows. When a star expands and surpasses the boundaries of its Roche lobe, some of its material enters a region where the gravitational influence of its companion star becomes stronger than its own gravitational pull. This initiates a process called Roche lobe overflow (RLOF). In this process, the material from the first star is pulled towards the companion star. This transferred matter can either be directly absorbed by the companion star through direct impact or it can form an accretion disc around the companion before eventually being accreted.



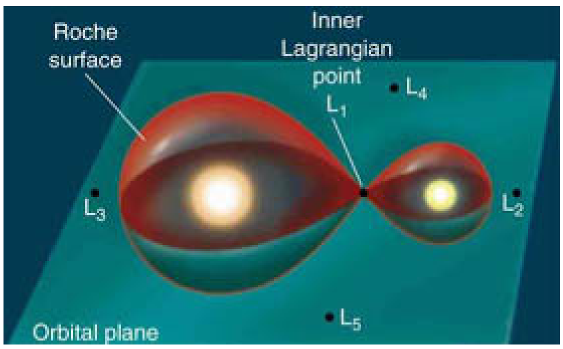


Figure 2.3 and 2.4 Roche Lobe OverFlow and Roche surface

2.3.2 Unstable Mass Transfer

Mass transfer becomes unstable when the accreting star is unable to absorb all the material being transferred from the donor star, as the accretor’s gravity is too weak to effectively capture the falling material. This surplus material accumulates on the accretor, causing it to expand. Eventually, the accretor's expansion leads it to fill and even exceed its own Roche lobe. As a result, a common-envelope (CE) system forms, in which the core of the donor star and the companion star are engulfed within the shared envelope of the donor star. This type of mass transfer generally occur when two massive stars are in close proximity.

Another cause of formation of a Common Envelope system is that the massive star which ran out of its fuel stars expanding and hence engulfing it’s companion star. There’s loss of angular momentum of the orbiting star due to the friction. The two possible outcomes from the CE phase is that either both star will merge in CE phase itself, or the Common Envelope will be shattered by stellar winds and thermal pressure, leaving just cores or stellar objects orbiting within more closer distance.

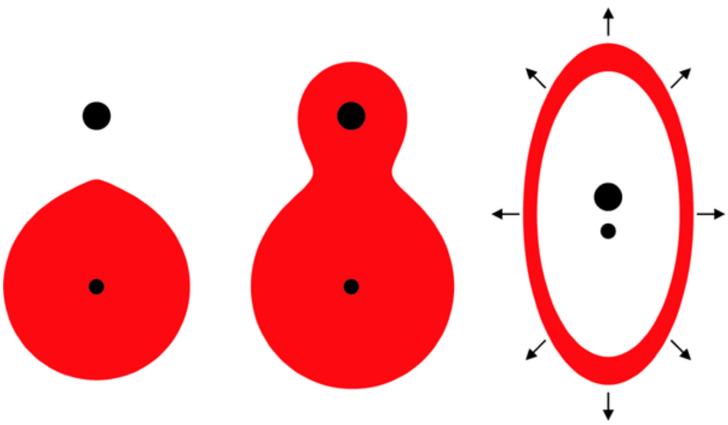


Figure 2.5 Evolution in Common Envelope phase

In some scenarios, the mass transfer are not conservative in nature. There is constant mass loss apart from burning nuclear fuel, by stellar winds. Thermal pressure and magnetic field are major causes of these winds. Whereas, in conservative mass transfer, all mass lost by the donor is accreted on the companion star.

As mentioned earlier, there are also some cases when stars go without any mutual interaction if the separation between them is too large.

Talking about cases, there’s a case when the less massive star in the binary system approaches its death quicker than the more massive star. This phenomena takes place when the more massive star while out growing its radius than the Roche Lobe surface suffers mass loss, and mass is transferred to the companion star, making it more massive, which causes it to burn it’s fuel more quickly. This is termed as Algol’s paradox, and it affects the evolution stages of star.

2.4 End of Binary Stars

Mass transfer in binary systems can lead to various outcomes, including the formation of different types of compact objects and the potential for mergers. The ultimate state of the binary system depends on several factors. Here are the possible scenarios for intact binaries where one or both stars have evolved into compact objects:

a) Dwarf Novae:
When the binary is composed of a white dwarf and a main sequence star, with the main sequence star being in close proximity, the material are being pulled of by the white dwarf. The mass transfer rates are so high that the accretion disk becomes very hot and luminous. Such stars are called ‘novalike’
If enough mass is built up to initial thermonuclear reactions, then the outer shells of white dwarf acts as a star’s core which is known as a ‘classical nova’.
.If due to accretion of mass on the white dwarf, the mass of it exceeds Chandrasekhar limit, then it undergoes a supernovae, making the white dwarf a compact object such as a neutron star or a black hole.

b) X-ray Binaries:
When one of the companion of a main sequence star is a compact object in the binary system, then due to intense gravitational pull, the material in accretion disk reaches to very high temperature and pressure which causes the accreting material to emit X-rays. Depending on mass of donor star, these are further classified into high mass X-ray binaries and low mass X-ray binaries.

c) Both of them being White Dwarfs:
If both stars in the binary system are not massive enough, they both end up being white dwarfs. Here, the less massive white dwarf spirals inward, gets disrupted by gravitational tidal forces, and its material either falls onto the more massive white dwarf or spreads into a broad disk.

d) White dwarf and a Compact object:
A system where mass has been transferred and the heavier star underwent supernovae. Here, in case of a WD+NH, the white dwarf experiences the tidal disruption. If the combined mass exceeds the Tolman–Oppenheimer–Volkoff limit (the maximum mass a neutron star can have before collapsing), the neutron star will collapse into a black hole. Or else, the white dwarf will simply merge into a black hole after tidal disruption, in case of a WD+BH.

e) Both being double compact objects:
Here, both of the stars have went through the supernovae.
In a NS+NS merger, both neutron stars gradually in-spiral inwards. Their merger will form a more massive neutron star, or if they crossed the Tolman-Oppenheimer-Volkoff limit, they'll form a blackhole.
A rare merger, which is between BH and NS, which results in formation of a larger BH and emission of powerful Gravitational Waves.
Even more powerful Gravitational Waves are formed by a BH+BH merger.

2.5 About Merging

As Einstein formulated in his theory of General Relativity that any massive accelerating object produces gravitational waves, so it is in case of orbiting stars or dwarfs or compact objects. These ripples through space-time is observed here on earth by LIGO (Laser Interferometer Gravitational wave Observatory), with simple principle of interference of light. We'll now look about phases merging

In the Pre-Inspiral phase the binary system might experience a period of relatively stable orbits where gravitational radiation is negligible. This phase can last for millions or billions of years before significant gravitational wave emission starts.

During the Inspiral phase, Initially, the binary system's orbit slowly shrinks as it releases weak gravitational waves. This phase lasts a long time, with the orbital speed and gravitational wave frequency gradually increasing. The inspiral phase continues until the system reaches the innermost stable circular orbit (ISCO).

Once the inspiral phase concludes, the compact objects plunge towards each other, resulting in a merger. This phase is marked by a peak in gravitational wave emission as the objects collide and merge into a single entity.

After the merger, the newly formed single black hole enters the ringdown phase. During this stage, the black hole "rings" or oscillates, emitting gravitational waves that gradually diminish. The ringdown phase starts when the black holes merge and the resulting black hole stabilizes within the photon sphere, where light is so strongly bent that it loops around.

Following the ringdown, the black hole continues to emit low-level gravitational waves as it settles into a stable state. This phase may include the emission of electromagnetic radiation if any residual matter is accreted by the black hole.

Now, lets have a look on Merger Methods:

a) Isolated Binary Evolution via CE phase -

Lets consider a case where two massive stars (100 $M_\odot$ and 75 $M_\odot$) form in a low-metallicity environment at a separation of about 10 AU. The more massive primary star evolves first, expanding and transferring mass to the secondary star, which causes the system to widen to around 20 AU.

The primary eventually loses its envelope, becoming a Wolf-Rayet star, and collapses into a black hole. Later, the secondary star undergoes a similar process, expanding and transferring mass to the black hole, leading to a shrinking orbit.

This results in a common envelope phase, where rapid mass transfer causes the black hole to spiral inwards, expelling the envelope. This leaves a black hole-Wolf-Rayet binary with a separation of approximately 35 $R_\odot$. Eventually, the secondary star also collapses into a black hole, forming a binary black hole system. This binary will merge in about 10 billion years due to gravitational wave emission.

b) Chemical Homogeneous Evolution -

Binary companions can raise tides on each other, and if close enough to fill significant portions of their Roche lobes, tidal energy dissipation leads to tidally locked rotation periods synchronized with their orbital period, rotating at significant fractions of their break-up velocities.

This rapid rotation induces temperature gradients between the poles and equator, causing large-scale meridional circulation and efficient mixing of hydrogen into the core and helium into the envelope.

As the stars reach the end of the main sequence, they behave like Wolf-Rayet stars, contracting rather than expanding. If the metallicity is low enough to prevent significant wind-driven mass loss, the binary can avoid mass transfer and maintain co-rotation and chemically homogeneous evolution.

c) Dynamical Evolution in a dense stellar environment -

The merging black holes might not originate from the same binary but instead form independently and meet in dense stellar environments like globular clusters.

As the most massive objects, they migrate to the cluster's center through dynamical interactions, where they form binaries via three-body interactions or by replacing lighter stars in existing binaries. If the binary is "hard" (with higher orbital speeds than typical stars in the cluster), further interactions with other stars tighten the binary, removing energy and gradually bringing the black holes closer together.

If the cluster density ensures enough interactions, the binary hardens until it merges through gravitational-wave emission, unless a recoil kick from the last interaction ejects it from the cluster, in which case it may still merge outside.

III

TASKS

3.1 Task #0 - Introduction to simulation and conservative mass transfer 26

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3.1 Task #0

Run 10 binaries, with only fully-conservative mass transfer (i.e all mass that is lost by a donor star during an interaction is accreted onto the companion star - none is lost to the environment). Compare the initial and final masses of both stars in each binary. Is the sum the same at the beginning and the end? If not, why not?

The BASH code for this task is as followed:
./COMPAS -–number-of-systems 10 -–mass-transfer-accretion-efficiency-prescription FIXED -–mass-transfer-fa 1 --mass-transfer-jloss 0 –detailed-output

For comparing initial and final masses, one can conclude that system's mass would not remain the same, mainly due to stellar winds. Also, supernovae can be a case where the star would expel out it's mass from the system. Here, lets look up in more detail for two of the cases out of ten.

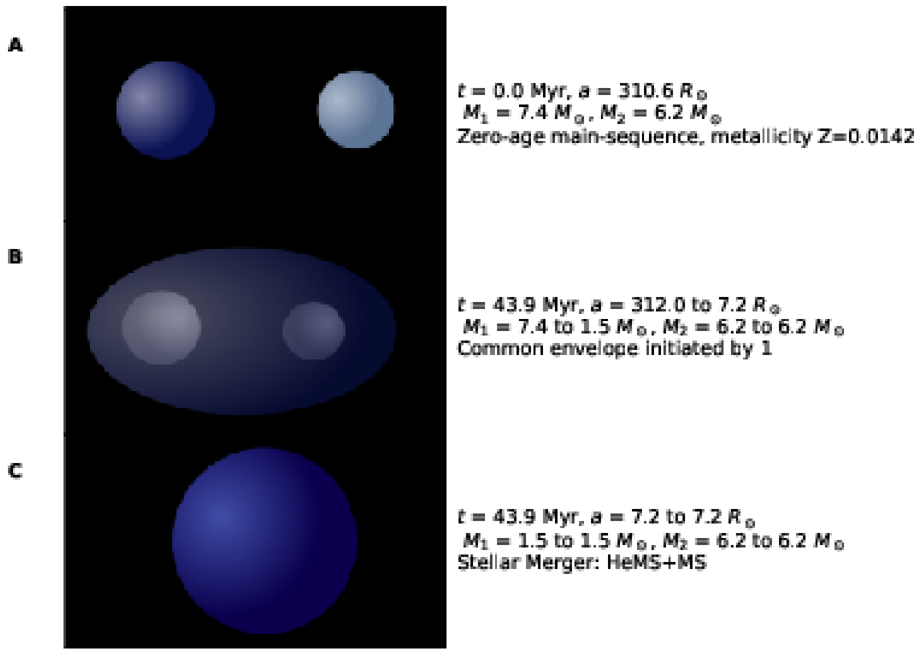


Figure 3.1 Van Den Heuvel plot of evolution of 1st binary system

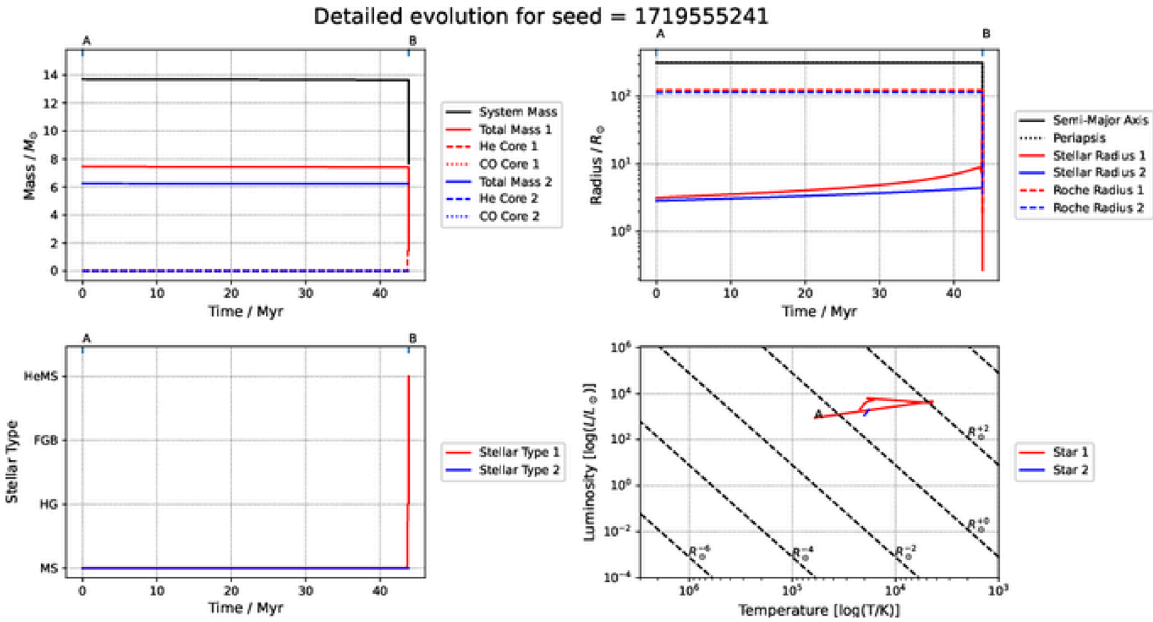


Figure 3.2 Statistical plot of various parameters stars in 1st binary system

We can observe, there's formation of a Common Envelope, followed by merger of a Helium Main Sequence which was evolved from the heavier Main Sequence star, and the other Main Sequence star. The total mass of system is constant, but there's very slight dip in it. It is caused by conversion of mass into energy by burning of fuel and stellar winds. Let's look into the second case:

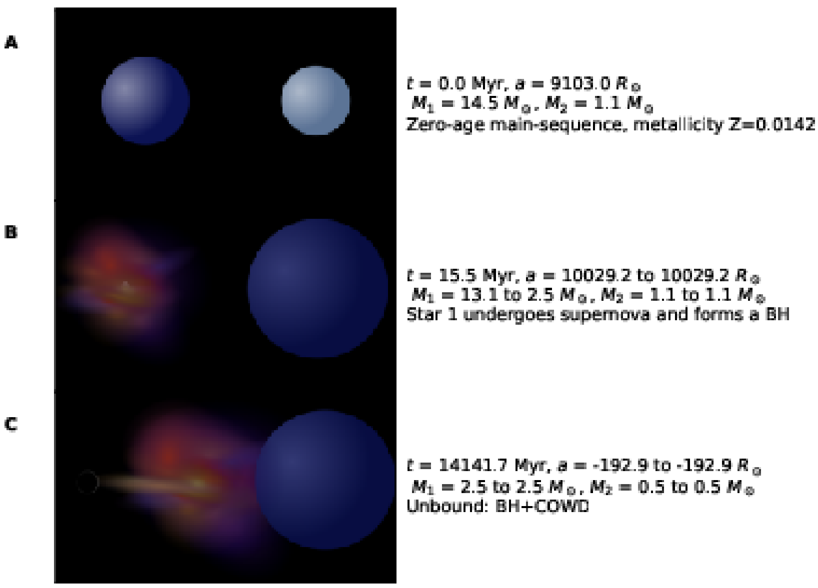


Figure 3.3 Van Den Heuvel plot of evolution of 2nd binary system

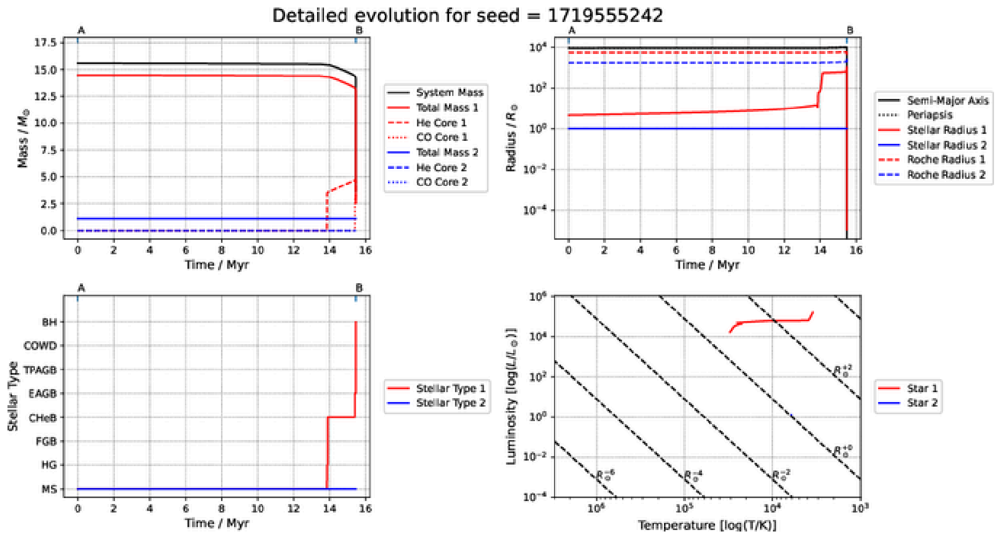


Figure 3.3 Statistical plot of various parameters stars in 2nd binary system

From the statistical plot, one can observe steady decrease of mass of the more massive star at around 14 Myr, due to its evolution from main sequence to Core Helium Burning phase. One can see sudden increase in its radius due to it passing through the giant branch phases. Steady mass loss can be explained by mass loss of outer expanding layers of star due to thermal pressure. At 15.5 Myr, 1st star went to supernovae which causes huge mass loss from the star.

3.2 Task #1

Run a single binary with detailed evolution turned on. Plot the evolution with the detailed evolution plotter. Be prepared to show the plot you created, and describe the major events in its life.

Bash code for this task is as follow:
./COMPAS --number-of-systems 1 --detailed-output

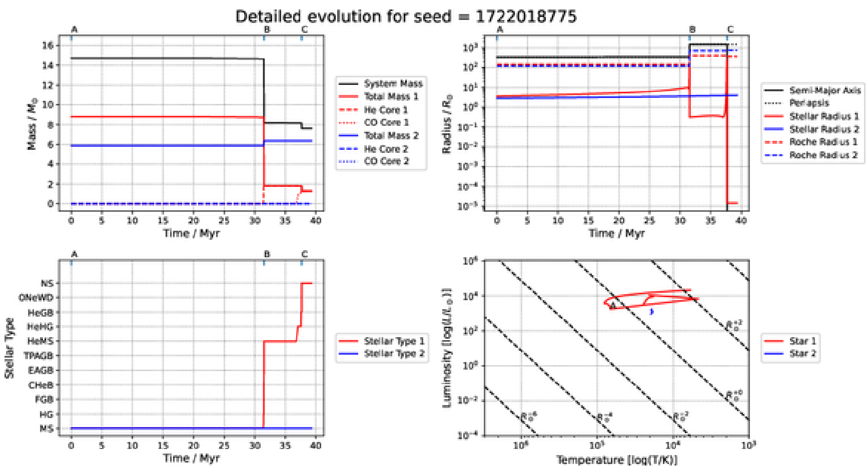
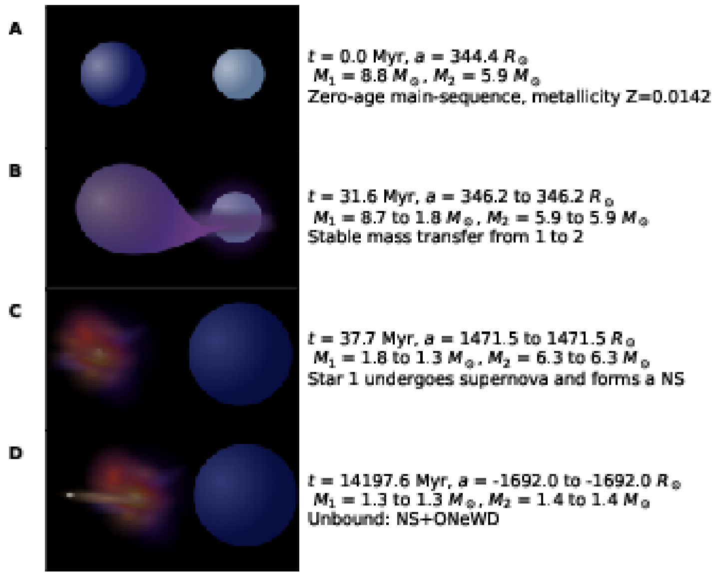


Figure 3.4

Statistical and Van Del Heuvel plot for Task #1



In this case, one can observe that 2nd star remains in the Main Sequence phase, and its radius, although being very unnoticed in graph, is increasing at a near constant rate. Due to stable mass transfer from Star 1 to Star 2 at around 32 Myr, Star 2 becomes more massive.

Let's now have a look on the 1st star, which underwent a lot of changes down the timeline. As it was more massive, it evolved a lot quicker than its companion. It was in it's Main Sequence phase until about 32 Myrs. It's radius increased at an non uniform rate, followed by an abrupt increase in its radius due to it entering in Giant Branch phases. It quickly evolves though the variability phases, and enters Helium Main Sequence phase, where burns Helium in its core. Notice how it quickly evolved through all the intermediate 'horizontal branches'. The Helium burning initiated from the point when a 'Helium flash' occurred and it entered Core Helium Burning phase. Then due to equalization of thermal pressure and gravitational forces, it shrink a bit in radius. Then followed by Helium Shell burning phase, which is similar to Hydrogen Shell burning phase, but the distinction is in the element which is being fused and formed, along with higher luminosity and temperature. For very short period in comparison to other phases, it remains in Helium Giant Branch phase, and then it undergoes Supernovae. It's also worth noticing how CO core grew it's mass when Helium Shell was being burnt and core was nourished with elemental Carbon and Oxygen. It seems phenomenal the way star 1 variably grew its outer radius in HeGB phase, and it quickly dropped after it becoming a Neutron Star due to a Supernovae, along with a drop in Semi Major Axis.

3.3 Task #2

Run 10,000 binaries and record how long it took. From the output, determine the fraction of these binaries that (1) Never interact (2) experience unstable mass transfer (or Common Envelope Evolution) (3) experience only stable mass transfer (4) undergo a stellar merger. These will be your dominant channels, though there may be some others - can you identify these? What fraction of your systems experience 0, 1, or 2 supernova explosions, and what kinds of explosions do you see?

Bash code for this task is: ./COMPAS --number-of-systems 10000

Figure 3.5 Scatter plot of various types of interactions of binary

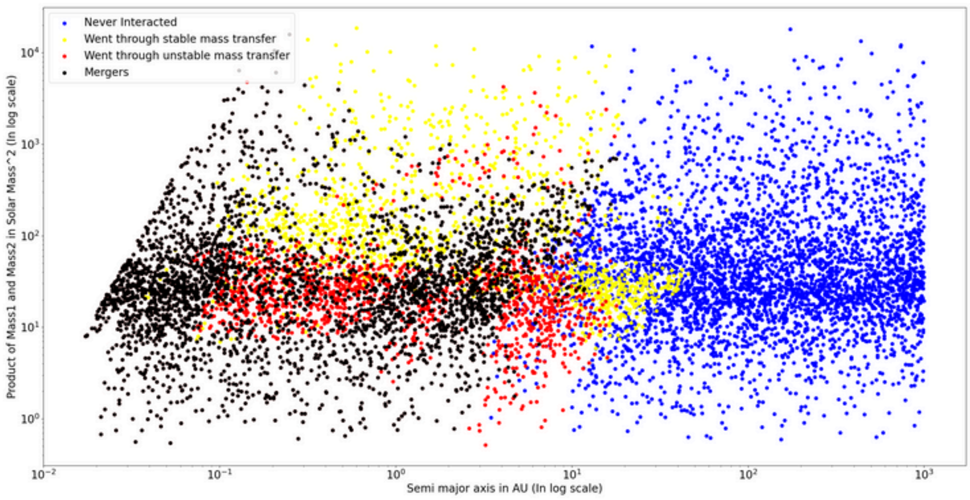
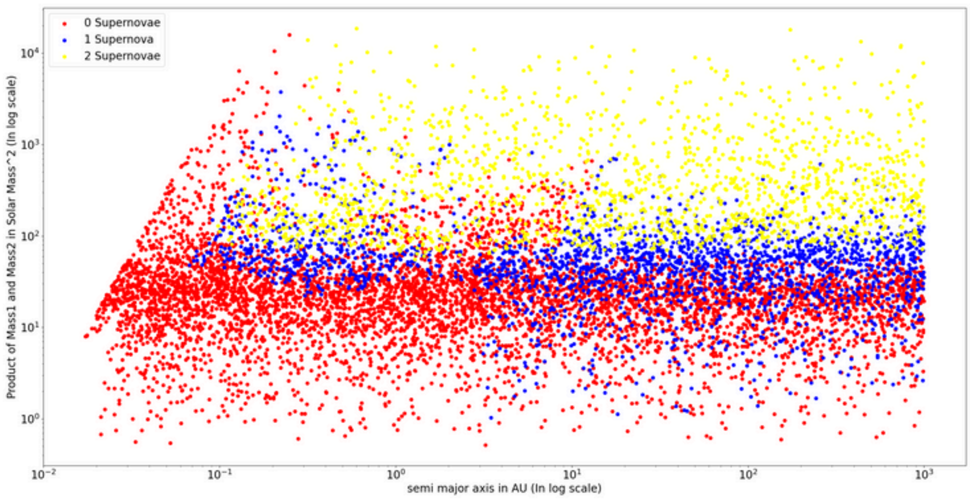


Figure 3.6 Scatter plot of number of times star went for a supernovae



NOTE: Mass Transfer = MT, RLOF = Roche Lobe Over Flow

I've used product of masses of star, as it seemed better parameter, rather than plotting two plots. First plot is about type of MT, no interaction and mergers.

1. There's clear distinction of binary system having no interaction. Those are systems having large semi major axis, or they are very distant, thus not interacting.
2. For stable mass transfer, there are two major areas, one having bunch of binaries having moderate distance and moderate mass, another bunch being moderate to hugely massive and lesser distant. I'm not sure about the first bunch here, but the second bunch having enough gravitational pull, and they being large enough to cross the equipotential surface causing RLOF.
3. For unstable mass transfer, stars to the side of being lesser massive. They don't have strong enough Gravitational force for RLOF, but as soon as they start to expand in later phases, they form common envelope. This wasn't possible for the RLOF case, as they were already massive enough for crossing the Roche Lobe.
4. For mergers, there are predominantly 2 major areas. One being shifted to more right, is depicting the case of merger after MT, both stable and unstable, as the cases are being overlapped there. The area on the left side shows the case where mergers are mostly happening in those binary having CE phase. Maybe they're being merged before stars become dwarfs or compact objects. Not sure why there's a gap in between those two areas.

If there's case of RLOF, that case won't be counted in unstable MT. If no RLOF occurred and there's a CE phase, those are counted to be in the group of unstable MT. Merger can happen in any MT cases. For the supernovae plot, there's clear layer wise distinction for number of supernovae. One with heavier combined mass had 2 supernovae, below it is the case of one, followed by zero supernovae.

Comparing both images, the abrupt slant red strip in 2nd image resembles the rightmost merger strip of 1st image in shape. Here, it may happen that stars in that particular case may have merged before forming any dwarf or compact object, and process of merger caused heavy mass loss, which resulted in no supernovae at later stages.

The above description was for the scatter plot, which gives us a brief idea that how binary systems in larger numbers evolve and what percentages of binaries undergo such interactions. After this visual temptation, lets have a look on actual numbers.

Time it took to simulate 10,000 binaries = 87.7017 CPU seconds
Total Number of Common envelopes formed: 4971
Total Number of Mass transfers: 15958
Total Number of double compact objects formed: 426

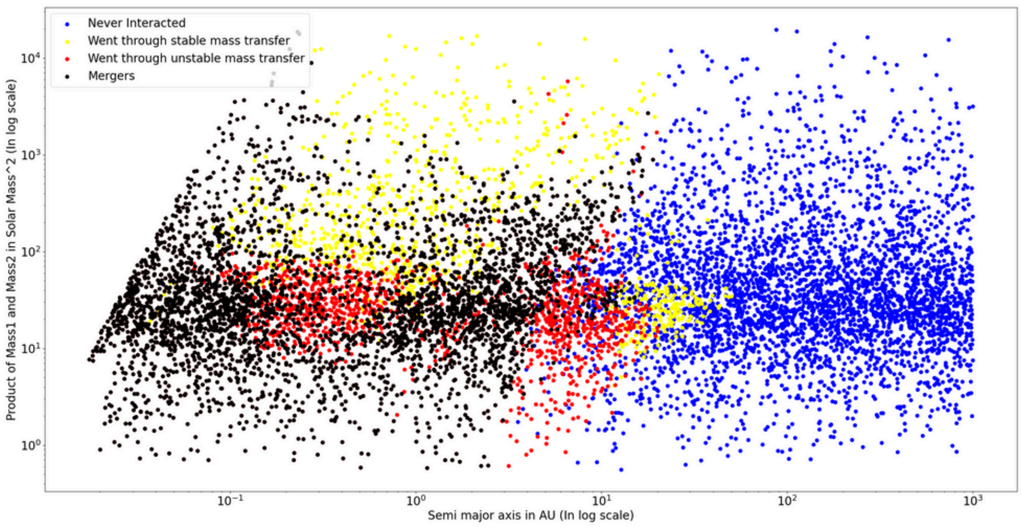
Number of Non-interacting binaries: 4087 (Fraction = 0.4087)
Number of binaries with stable mass transfer: 1356 (Fraction = 0.1356)
Number of binaries with unstable mass transfer (CEE): 4557 (Fraction = 0.4557)
Total number of stellar mergers: 3142 (Fraction = 0.3142)

Total Number of Supernovae: 5447
Number of stars with 0 supernovae: 6192
Number of stars with 1 supernova: 2169
Number of stars with 2 supernovae: 1639

3.4 Task #3

Repeat the 10,000 binary run, but with fully conservative mass transfer turned on again. How do the fractions of interacting and merging systems change? Are the initial conditions identical to the previous run? What could you do to make sure they are identical (i.e to make sure that the only differences come from the assumed physics)?

Bash code for this task is: ./COMPAS ––number-of-systems 10000 ––mass-transfer-accretion-efficiency-prescription FIXED ––mass-transfer-fa 1 --mass-transfer-jloss 0



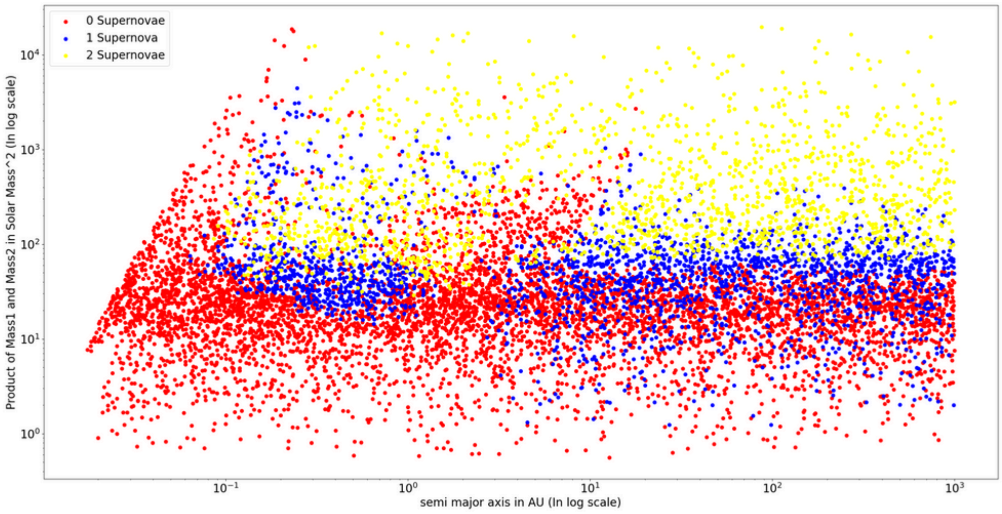


Figure 3.6 Scatter plot of number for type of interaction and supernovae respectively

Scatter plot for type of interaction and number of supernovae shows almost similar trends for both conservative and non conservative cases, except some minor deflections. Now lets look at on number, followed by histogram showing trend fir mass lost with respect to number of binary system for both cases. For making sure that initial conditions are similar, prefer using '--random-seed ' to make sure that initial seed for both cases are same and consequently, all seeds are same.

	Conservative	Non Conservative
Time it took to simulate 10,000 binaries (In seconds):	83.9189	87.7017
Total Number of Common envelopes formed:	5508	4971
Total Number of Mass transfers:	15494	15958
Total Number of double compact objects:	486	426
Number of Non-interacting binaries:	4079	4087
Number of binaries with stable mass transfer:	994	1356
Number of binaries with unstable mass transfer (CEE):	4927	4557
Total number of stellar mergers:	3643	3142
Total Number of Supernovae:	5513	5447
Number of binaries with 0 supernovae:	6143	6192
Number of binaries with 1 supernova:	2201	2169
Number of binaries with 2 supernovae:	1656	1639

In the above comparison, first lets talk about the case where we can see negligible changes. Number of non-interacting binaries and number of binaries with supernovae are almost similar in both cases, as, both of these fundamentally depend on initial conditions, which was set to be similar to compare the conservative and non conservative case.

Initial distance being a major contributor for interacting binaries, was same in both cases for all binaries which caused almost similar number showing up for non interacting binaries, despite of the fact that one of the case was conservative. Although the slight decrease of 8 binaries which didn't interacted might be the one that would be on verge of interaction by a small push of differing parameter.

Similarly, supernovae primarily depends on initial mass of star. Other factors such as mass transfer, and mass loss do affect the number of supernovae, which can be observed as slight increase of 66 more supernovae in the conservative case, which hence caused larger number of Double Compact objects to form.

Now lets talk about the cases having noticeable distinctions. There are 537 more common envelopes formed in conservative case. The continuous addition of mass to the companion star in a conservative scenario often destabilizes the binary system, leading to Roche lobe overflow, orbital shrinkage, and ultimately the engulfing of both stars in a common envelope, which might not happen in the other case.

Number of mass transfers is more in non conservative case by 464. Due to abrupt mass loss by stellar winds and loss of angular momentum, one can expect change in orbital parameters to be more frequent. Also, there are more Roche Lobe reconfigurations due to this, and hence, one can observe more mass transfers in non conservative case. There's a shear contrast in stable and unstable mass transfer. This clearly explains why conservative case had more unstable mass transfer while converse is true for non conservative case.

Conservative case also leads by the number 501 from the non conservative case for number of stellar mergers. Common Envelope phase is in direct relation with stellar mergers. More the Common Envelopes, more will be mergers, as there's a lot of complex interaction going in CE phase, which consequently bring two stars more closer. Also, double compact objects formed in conservative case is more by 60. As there's no mass loss outside the system, it ultimately make stars in later stages to cross the Chandrasekhar limit and hence forming a DCO.

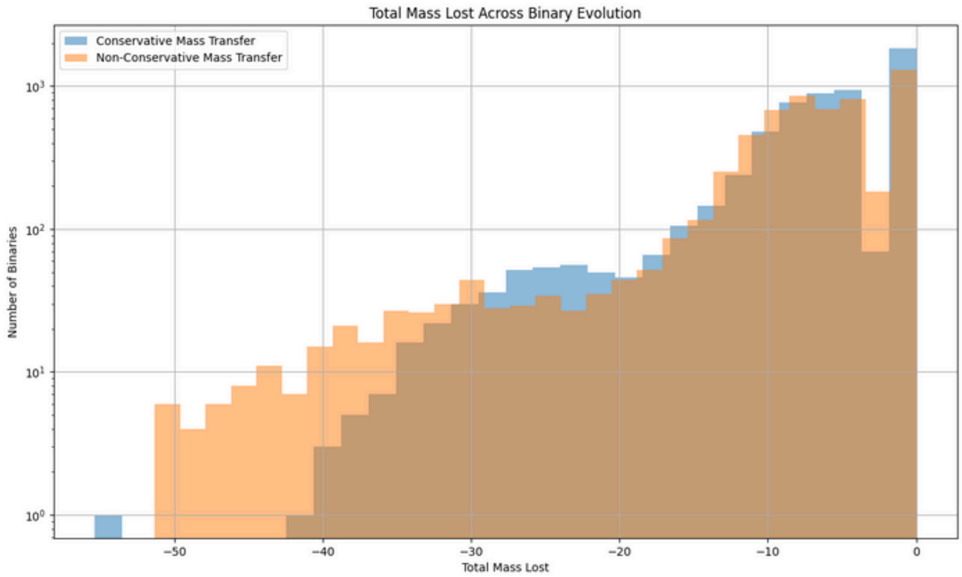


Figure 3.7 Histogram for mass lost during mass transfer event vs number of binaries for conservative and non conservative cases

For a large range of mass loss values, both conservative and non-conservative cases show significant overlap, with a large number of binaries losing similar amounts of mass. On the left side of the plot (where mass loss is more negative), non-conservative mass transfer has more systems losing a large amount of mass compared to the conservative case, which matches our assumption of no mass loss due to external factors. However, at certain ranges of mass loss (e.g., between -30 and -10 solar masses), the conservative case shows more mass loss compared to the non-conservative case. This is reflected by the blue bars that rise above the orange ones in those regions.

The observed higher total mass loss in some ranges suggests that the system experiences multiple phases of mass transfer where significant mass is cycled between the stars. This could lead to an accumulation of mass loss events, with each mass transfer reducing the total mass of the binary as the transferred mass might not be perfectly conserved over multiple events due to complexities like stellar winds or eventual mass ejection later in the binary's evolution.

The system may go through extended or multiple phases of mass transfer that cumulatively result in more mass being lost by the donor over time, leading to higher total mass loss recorded for conservative cases in those ranges. Another reason is, conservative mass transfer might have created some scenarios where star may undergo through some phases where it losses it's mass in bulk. Example here being, there are more supernovae in conservative cases, which explains mass loss in some cases. Another example being stars may evolve to some more complex giant branches and losing mass through shedding layers.

3.5 Task #4

Run 100,000 binaries, in parallel. This can be done using a simple for loop in a bash script, creating background processes. If you don't know what this means, please reach out and we can help you get setup with this. If you have 10 free cores on your computer, you can run these 100,000 binaries in the amount of time it took you to run the 10,000 in the previous step. Verify that this is the case, and then combine the output files into one larger output. From this larger output file, how many Binary Black Holes do you produce? How many of these will merge in a Hubble time? How many Binary Neutron Stars, and how many Black Hole - Neutron Stars do you find, and how many of these merge within a Hubble time?

Bash code for this task is:
nano run_parallel_COMPAS.sh

```
  if [[ $end -gt $total_systems ]]; then
    end=$total_systems
  fi

  # Define output file for each batch
  output_file="$output_dir/COMPAS_Output_batch_${batch_num}.log"
  error_file="$output_dir/COMPAS_Output_batch_${batch_num}_error.log"

  # Run COMPAS
  echo "Running batch $batch_num from $start to $end with $batch_size systems"
  ./COMPAS --number-of-systems $batch_size >"$output_file" 2>"$error_file"
}

export -f run_COMPAS
export total_systems
export batch_size
export output_dir

# Number of batches needed
num_batches=$(( (total_systems + batch_size - 1) / batch_size ))

# Run the simulations using GNU parallel
seq 1 $num_batches | parallel -j $parallel_jobs run_COMPAS

echo "All simulations are done."

./run_parallel_COMPAS
```

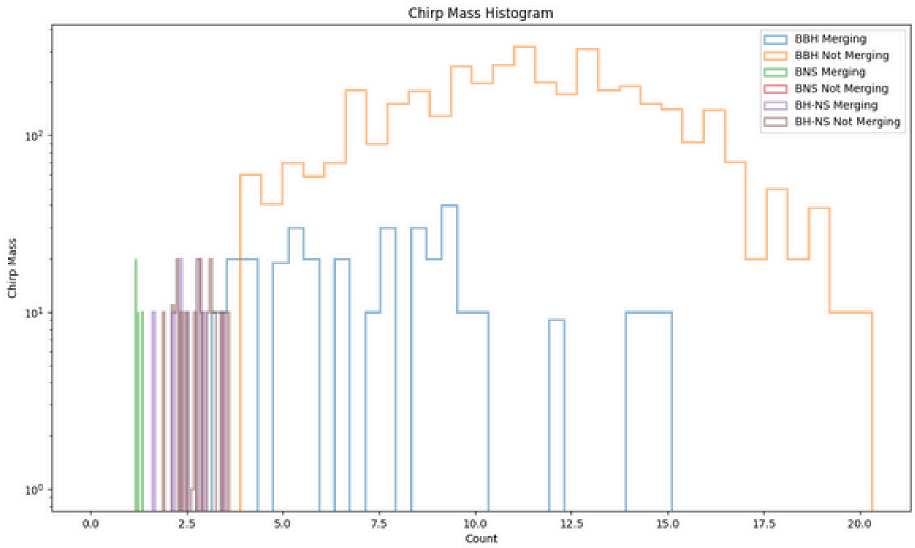


Figure 3.8 Histogram for the count of different merging and non merging double compact objects versus their chirp mass

Chirp mass is a specific combination of the masses of two orbiting bodies (like stars or black holes in a binary system) that plays a crucial role in the dynamics of their inspiral and the gravitational waves they emit. It is particularly significant in the context of gravitational wave astronomy.

The chirp mass M is defined by this equation:

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

High Chirp Masses Leading to mergers of binary black holes (BBH) systems can be explained as higher chirp mass means these binaries emit stronger gravitational waves and lose orbital energy faster, leading to quicker mergers. However, there are even more massive BBH systems which do not merge in Hubble time over a wide range of count. The intuitive explanation of high initial separation supports this outcome.

The green bars show merging Binary Neutron Star (BNS) systems, which have much lower chirp masses compared to BBHs because neutron stars are less massive than black holes. These systems are still compact enough to merge, but they are clustered at the lower end of the chirp mass spectrum. These lower chirp masses mean the binaries inspiral slower than BBHs, but they are still within the range where merging is expected within a Hubble time. The pink bars represent non-merging BNS systems. They have similarly low chirp masses, but these systems are either in wider orbits or have dynamics that prevent them from merging within a Hubble time. Here in our case, we observe no such cases where BNS system do not merge.

BH-NS Merging: The brown bars show merging BH-NS systems. These binaries have a range of chirp masses, which is expected given the varying mass ratios between black holes and neutron stars. The presence of both higher and lower chirp masses reflects the diversity in mass combinations within these systems. These systems tend to merge because the black hole's strong gravitational influence pulls the neutron star into a rapid inspiral. The purple bars show non-merging BH-NS systems. They are fewer in number and typically have chirp masses that do not favor quick mergers. This could be due to wider initial separations or configurations that make the loss of orbital energy slower.

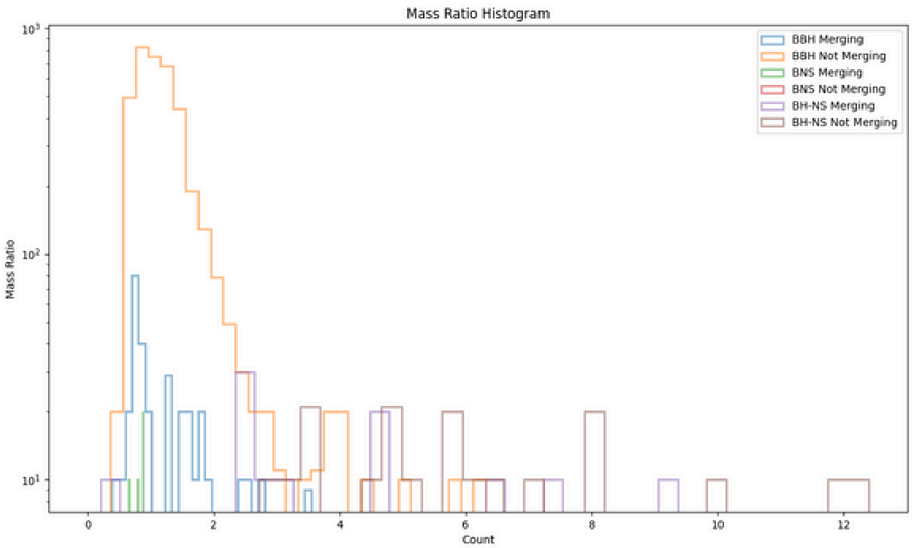


Figure 3.9 Histogram for mass ratio of different merging and non merging double compact objects versus their count

The BBH systems having higher mass ratio to the left (lesser in number) is not likely to merge in Hubble time. This is due to ineffective gravitational waves being produced due to such asymmetrical ratio, which results in low orbital energy emission. The one having lesser mass ratio and still not merging is due to larger initial separation. Merging BBH systems thus have lower mass ratio, which indeed can be observed from the plot.

Merging BNS Systems with Consistent Masses ($q \approx 1$): Neutron stars have a relatively narrow mass range (typically around 1.2 to 2.1 solar masses), so most BNS systems have mass ratios close to 1. This leads to efficient gravitational wave emission and a higher likelihood of merging. BNS systems often form in close orbits due to the dynamics of their progenitor stars. The short orbital period and consistent mass make these systems prime candidates for mergers. Although the mass ratio is conducive to merging, some BNS systems might have stable, wide orbits that prevent significant orbital decay over a Hubble time. These systems might have experienced less dynamic interactions or supernova kicks that kept them at a safe distance.

In the BH-NS systems, the black hole is significantly more massive than the neutron star. The gravitational pull of the black hole can lead to rapid orbital decay, especially if the neutron star is close enough to spiral inward due to gravitational wave emission. If the separation is significant, loss of orbital energy overcome the effect of high gravitational pull of Black hole resulting in no merger in case of high mass ratio.

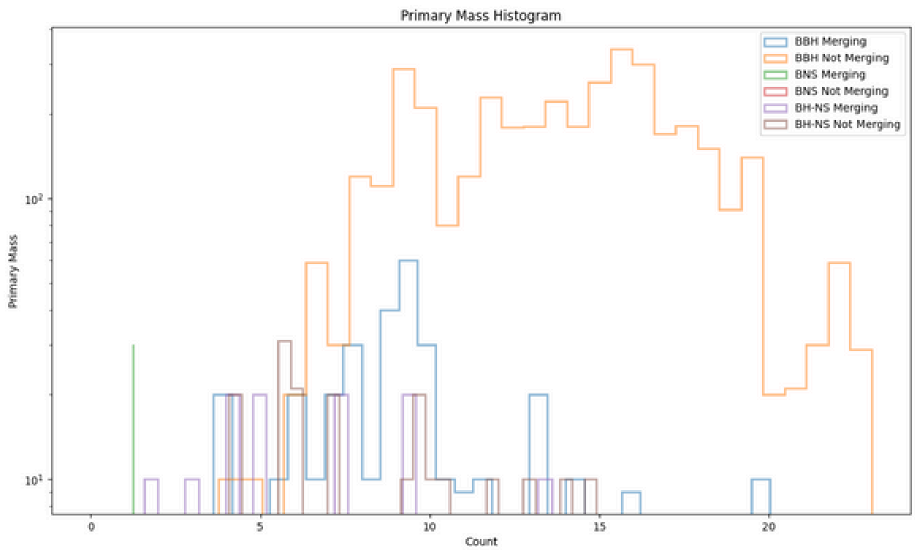
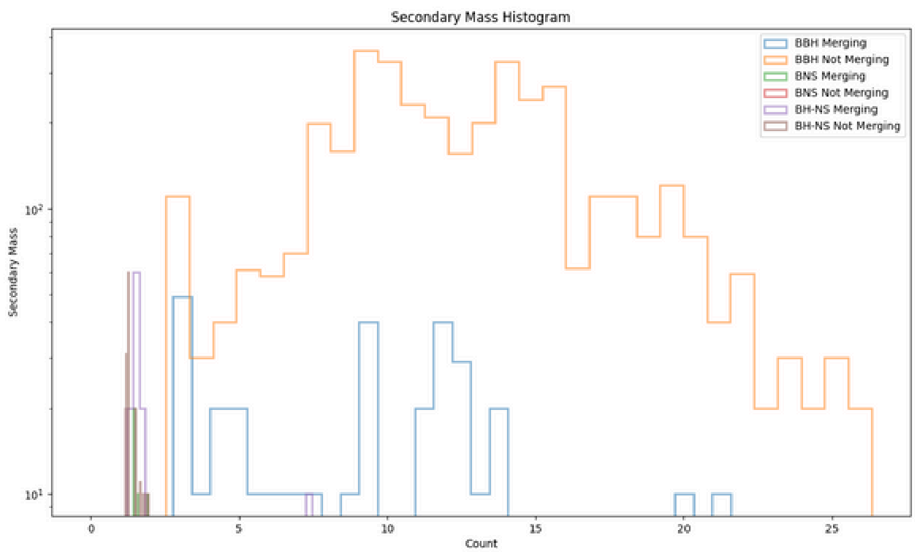


Figure 3.9 (a) Histogram for primary and secondary mass respectively versus count of double compact object



After running simulations in COMPAS and analyzing the resulting data, we have successfully gained valuable insights into the behavior of binary star systems under various conditions. The simulations have provided a wealth of information on key parameters such as mass transfer events, number of mergers, factors affecting mergers, and the impact of different evolutionary paths.

From plotting a scatter plot to know how things work in reality by picturing them in a larger number to comparing histograms of total mass loss and analyzing the distinctions between conservative and non-conservative mass transfer cases, we have been able to elucidate how these factors influence binary evolution.

This comparative analysis not only enhances our understanding of stellar dynamics but also contributes to refining theoretical models and improving predictions for future simulations, even if it was just a basic extrapolation and interpretation of the simulations. The findings underscore the importance of detailed data analysis in uncovering the underlying processes governing binary star systems and pave the way for further exploration and validation of theoretical predictions.

IV

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