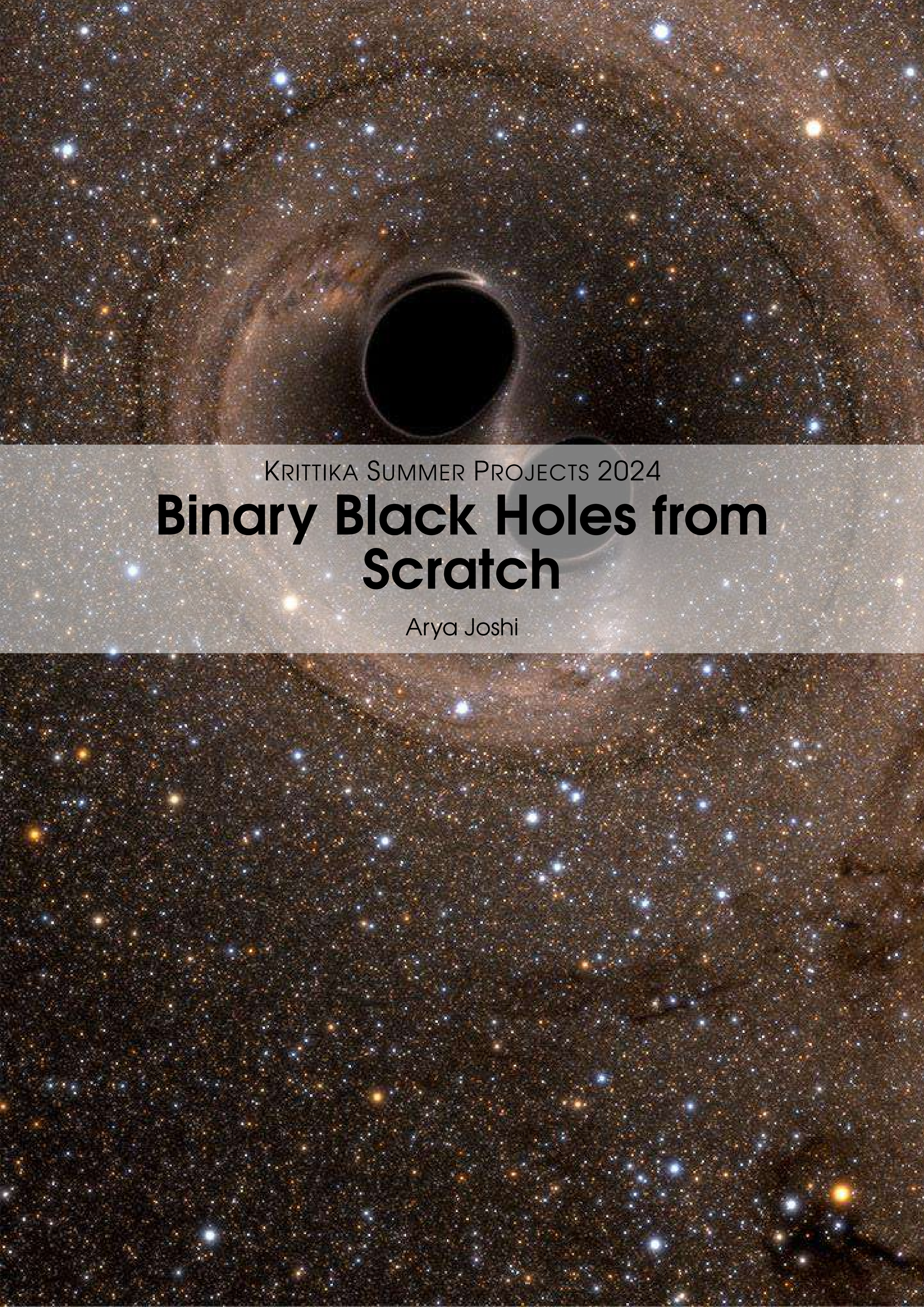

Krittika Summer Projects 2024

# Binary Black Holes from Scratch

Arya Joshi

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# Binary Black Holes from Scratch

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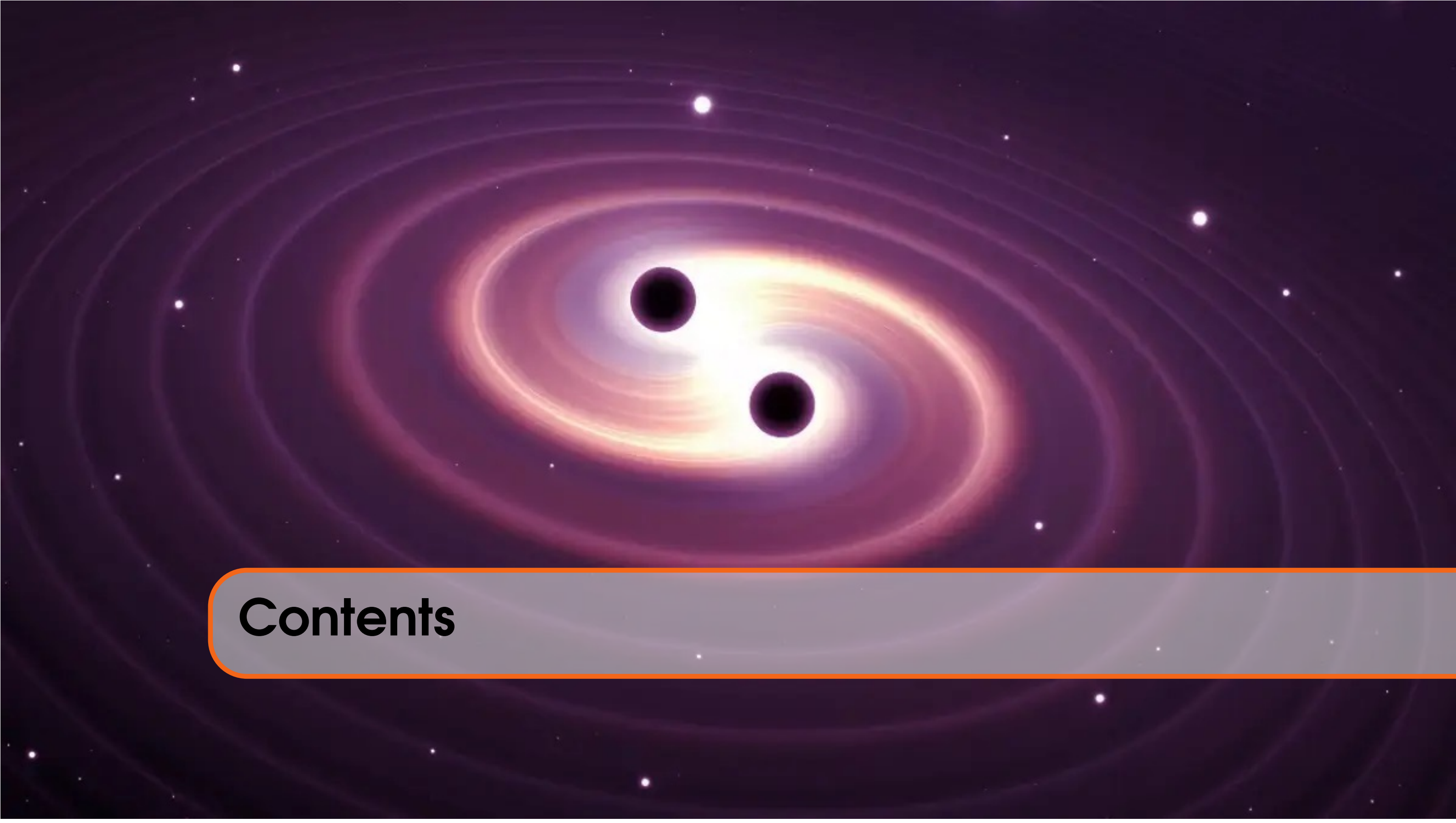
# Abstract

The lifetimes of massive stars are much shorter than our Sun's. However, if these stars happen to be in binary system, many interesting things can happen even in that short time. One of these is the formation of Compact Object Binaries, two black holes/neutron stars orbiting each other eventually merging into a bigger black hole, by radiating their energies in the form of Graviational Waves. The gravitational waves can be detected by observatories here on Earth, like LIGO and Virgo. In this project, we try to simulate these compact object binaries using COMPAS, a binary population synthesis code. After familiarising ourselves with COMPAS through different types of tasks, involving plotting obervations and realizing trends, we can simulate binaries and try to compare their data with that of actual gravitational wave data observed by the LIGO-VIRGO collaboration.



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# I

# Theory

# 1. Single Stellar Evolution

## 1.1 Introduction

The lifecycle of star is known as ‘Stellar Evolution’. In single stellar evolution, we track an individual star as it evolves from its birth till its death. Understanding this process is fundamental to understanding the dynamics in the universe, as stars are the most active components of present day universe.

## 1.2 Hertzsprung-Russel Diagram

Before we get to the life-cycle of stars, let us quickly understand something which we will be using frequently to track the life of a star, the Hertzsprung-Russell Diagram. When classifying stars, we want to use quantitative measurements, so that we can understand how, and why, stars differ from each other. There can be many physical characteristics of a star, which give different amounts of information about the lives of stars. However, by far, the two most important quantities of a star are its mass and age. The mass of a star is the defining quantity of how the future of a star will unfold. Age of a star determines how far along the star has come in its life. However, both these quantities are difficult to figure out from direct observation of stars.

Ejnar Hertzsprung and Henry Norris Russell discovered that plotting the brightness (which can also be related to absolute magnitude) of an individual star against its colour results in the stars lying in well defined regions on the graph. Further research showed that this plot was directly connected to the evolutionary status of the plotted stars.

In the figure given below, we can see that there are well defined regions in which stars lie. The majority of stars, which are core hydrogen burning, lie on the main sequence line. Stars that are beyond their main sequence lives lie on the giant branch or the super giant branch. All the dots indicate individual stars and their position on the plot gives an idea about their age and mass they must have to be in the spot that they are on the plot.

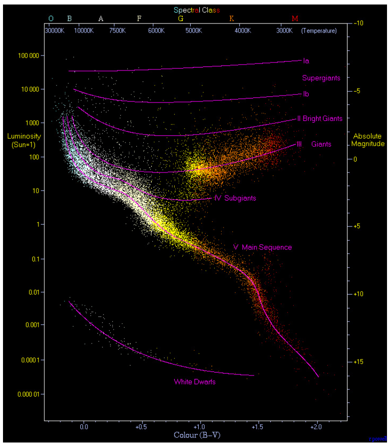


Figure 1.1: Hertzsprung-Russell Diagram

## 1.3 Stellar Birth

- Stars are formed in parts of space that are called star-forming regions. These star-forming regions are regions of concentrated gas and dust, collapsing under there own gravity to give birth to stars. This gas falling under its own gravity is what is called a **proto-stars**.
- The gas falls down the gravitational potential well as it is accreted onto the proto-star, leading to an increase in its kinetic energy. Due to the viscosity of the gas, it also starts heating up as it is accelerating. As it gets hotter, it gives light until it falls on the surface of the proto-star, where even more light is produced.

## 1.4 Main Sequence

- When the protostar has finished accreting all the gas and dust that it could, if the star is massive enough, it can start burning Hydrogen in its core. When this happens, the star becomes a zero-age main sequence (ZAMS) star.
- From here onwards, the star spends a majority of its life as a main sequence star, fusing hydrogen into helium in its core.
- The amount of time that a star will spend in the Main Sequence depends *inversely* on its mass.

$$\text{Lifespan of a star} \propto \frac{1}{\text{Mass of the star}}$$

1. Stars that have mass lesser than $0.6M_\odot$ (Solar Masses) are called as **Red Dwarfs**. They have lifespans of almost a hundred billion years.
2. Stars having mass comparable to our Sun have lifespan of around 10 billion years.
3. On the other hand, stars much more massive than the Sun, burn through their fuel supply extremely quickly. Many such stars have lifespans of only a few hundreds of millions of years.

- Stars in the main sequence do not have much variations from the outside. However a lot is going on inside the star. One way of studying what happens inside a star is by measuring the vibrations of the star's surface, called pulsations. This study comes under the **astroseismology**.

## 1.5 Beyond Main Sequence

- The point when a star burns through all of its hydrogen marks the end of main sequence for the star.
- A star goes through many changes as it leaves the main sequence. The most notable change among these is that it turns into a **Red Giant**, swelling up in diameter, with increased luminosity and a fall in its temperature.
- These stars are called the Asymptotic Giant Branch (AGB) stars, and they all have the same general interior structure, characterized by a carbon or oxygen core surrounded by layers of helium and hydrogen.
- Sometimes these AGB stars undergo **Thermal Pulses**, where the layer of helium suddenly undergoes thermonuclear burning. This causes drastic changes in the star's luminosity and temperature. This is also called as a **Helium Flash**.

## 1.6 Stellar Death

Once a star is past the AGB phase, its life is essentially at the end. However, what next is entirely dependent on the mass of the star at this point.

### 1.6.1 White Dwarf

- If the star exits the AGB phase with less than $1.4M_\odot$, it ends its life as a white dwarf. Stars which are initially many times more massive than our sun, may still end up losing mass during their lifetimes and having less mass than this limit by the end of their lives.
- The core of these stars is mostly comprised of carbon-oxygen, or oxygen-neon. The star sheds its outer layers forming a **planetary nebula**, with the core of the star remaining as a white dwarf.
- These cores are extremely dense, due to the extreme pressure from above layers, which was only repelled by electron degeneracy pressure.
- Red dwarf stars don't even swell up when they die. Since they comprise mostly of helium at the end of their lives, with some stars of relatively high mass being able to form carbon and oxygen in their cores.They quietly turn into white dwarfs.
- These white dwarfs will slowly continue to radiate their energy over trillions of years, only after which will they transform into their final form, **black dwarf**, a non-radiating body at thermal equilibrium with its surroundings.

### 1.6.2 Neutron Star

- If the mass of the star is more than $1.4M_\odot$ (also known as the **Chandrashekhar Limit**), the star keeps burning heavier elements in its core, fusing carbon and oxygen to form heavier elements.

- Eventually, these fusion reactions lead to the formation of iron. Beyond this, iron cannot be fused without needing more energy to fuse than what it releases. Therefore, iron is the limit of these nuclear fusion reactions.
- This results in an abrupt stop in the fusion reactions. The outer layers implode and bounce off the core, resulting in the one of the most energetic and violent events in the entire universe, **supernovae**.
- Through this gigantic explosion, only the core of the star remains, made entirely of neutron matter (the electrons and protons fuse to form neutrons), as a **neutron star**.
- These neutron stars are even more denser than white dwarfs, being held together by neutron degeneracy pressure.

### 1.6.3 Black Hole

- If the mass of the star is more than about $3M_\odot$, even neutron degeneracy pressure is not able to keep the core intact.
- The star implodes in a supernova, leading to the formation of the one of the most mysterious objects in the universe, **black hole**.
- Black holes are thought to have a singularity at their centers, covered up by an event horizon. Anything that crosses the event horizon cannot escape the black hole, not even light.
- Black holes are theorized to evaporate over a very, very long time ($10^{67}$ years for a black hole as massive as our sun), through a process called **Hawking Radiation**.

# 2. Binary Stellar Evolution

## 2.1 Introduction

So far we have looked at how a single, lonely star evolves on its own. However, most stars have been observed to be a part of binary or multiple star systems. The presence of another star in the vicinity of a star can greatly impact its trajectory of evolution. Here we see how stars in a binary system interact with each other, and what do these system produce as a result of these interactions, with special interest in how these systems can form **Double Compact Objects (DCO)**, i.e , pairs of black hole binaries, or neutron star binaries, or black hole + neutron star binaries.

## 2.2 Types of Binary Interactions

### 2.2.1 Roche Lobe Overflow

- In binary systems, there is a 3D boundary around each star where the gravitational forces of the stars are equal and opposite. This limit is called the **Roche Lobe**.
- If a star grows beyond this roche lobe, such as when it goes through its giant phase, it starts accreting some of its matter onto its companion star.

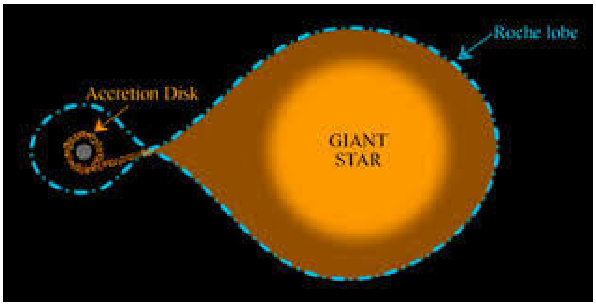


Figure 2.1: Roche Lobe Overflow

- Roche lobe overflow is considered to be a form of stable mass transfer, when all the mass lost by one star is accreted onto the other without any other disturbances.
- Many such binary systems have been observed, where the less massive star appears to have evolved more. Based the knowledge of SSE, this should be impossible. However in binary systems, this same star may have been more massive, leading to its quick evolution, then it could have started transferring its mass to its companion, through RLOF,leading to the observations that were made.

### 2.2.2 Common Envelope Evolution

- Sometimes, the two stars in a binary start sharing the same outer layers. This common outer layer is called as **Common Envelope**, and the binary is said to be undergoing common envelope evolution.
- This is type of unstable mass transfer, which may result from unstable RLOF.

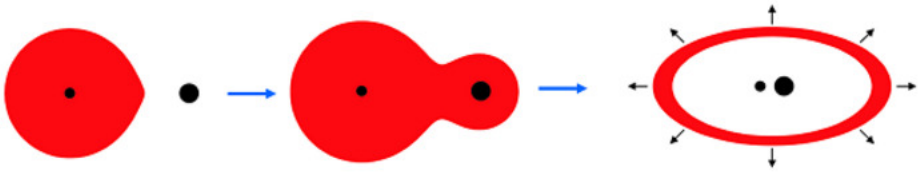


Figure 2.2: Common Envelope Evolution

- During common envelope evolution, the inner binary system is subject to drag forces, which causes the separation between the stars to decrease.
- This phase ends in either of two scenarios:
  1. The envelope is ejected out of the binary, with companion stars orbiting each other at a closer distance.
  2. If the stars are too close, the system undergoes a merger to form a single star.

### 2.2.3 No Interaction

- If the stars are small enough, or if they sufficiently space apart, the stars can undergo separate stellar evolution, without any influence from their companion stars.

## 2.3 Products of Interactions

### 2.3.1 Unbound Binaries

- Binaries can become unbound if one of the stars undergoes supernova with a high enough *Super Natal kick*, such that the core is ejected from the binary.

### 2.3.2 Systems with one compact object

- In these systems, one of stars is massive enough that it undergoes its evolution quickly and forms a compact object (neutron star or black hole), while the other star still remains in its main sequence.

- Usually the companion star is not massive enough to undergo a supernova, and instead the system forms a neutron star/black hole + white dwarf system.
- An interesting case of these systems are **X-ray Binaries**. In these systems, the main sequence star starts donating its mass to the compact object, which starts accreting on the neutron star/black hole. The infall matter releases its gravitational potential energy (upto 30% of its rest mass energy), as X-rays.
- In a somewhat similar case, if a binary comprises of a white dwarf and main sequence star, the star could begin accreting its mass on the white dwarf (RLOF). As a result, if the white dwarf crosses the mass threshold, it will implode and result in a supernova. These types of supernovae are categorised as **Type 1a supernovae**.

### 2.3.3 Double Compact Object Systems

- These are systems we are interested in the most. These types of systems are also relatively rare to form.
- Black hole/Neutron star binaries eventually merge, by radiating energy in the form of gravitational waves, which can be observed here on Earth (LIGO).
- In the next section, we discuss formation scenarios for such binaries which have *Coalescence Time* (time in which the binary merges) less than the present age of the universe (**Hubble Time**), since we have been able to detect some of these mergers in this time.

## 2.4 Formation Scenarios

### 2.4.1 Coming closer later in life

- This scenario imagines two high mass stars being born in a binary.
- The higher mass star reaches the end of its main sequence earlier, where it has completed fusing hydrogen into helium in its core.
- The star expands, overflowing the Roche Lobe, where it begins transferring its outer layers to other star. It loses its entire envelope and becomes a naked helium star, also known as **Wolf-Rayet star**. The star collapses into a black hole, and it may have some associated super-natal kick.
- When the second star reaches the end of its main-sequence, it overflows its roche lobe, and starts accreting its mass on the black hole. This mass transfer from higher mass donor to lower mass accretor requires the binaries to come closer.
- Due to dynamically unstable mass transfer, a common envelope is formed. After the expulsion of this common envelope, the two objects come even closer.
- The second star collapses and a black hole binary is formed.
- This process can take a few hundred million years, but the time required for the merger after this is nearly 10 billion years.

### 2.4.2 Chemically Homogeneous Evolution

- In this scenario, the companion stars raise tides on each other, like Moon's tide on the Earth. The tidal energy is dissipated rapidly, until the stars become tidally locked with each other. The rotation periods of the stars become synchronised to the orbital period of the binary.

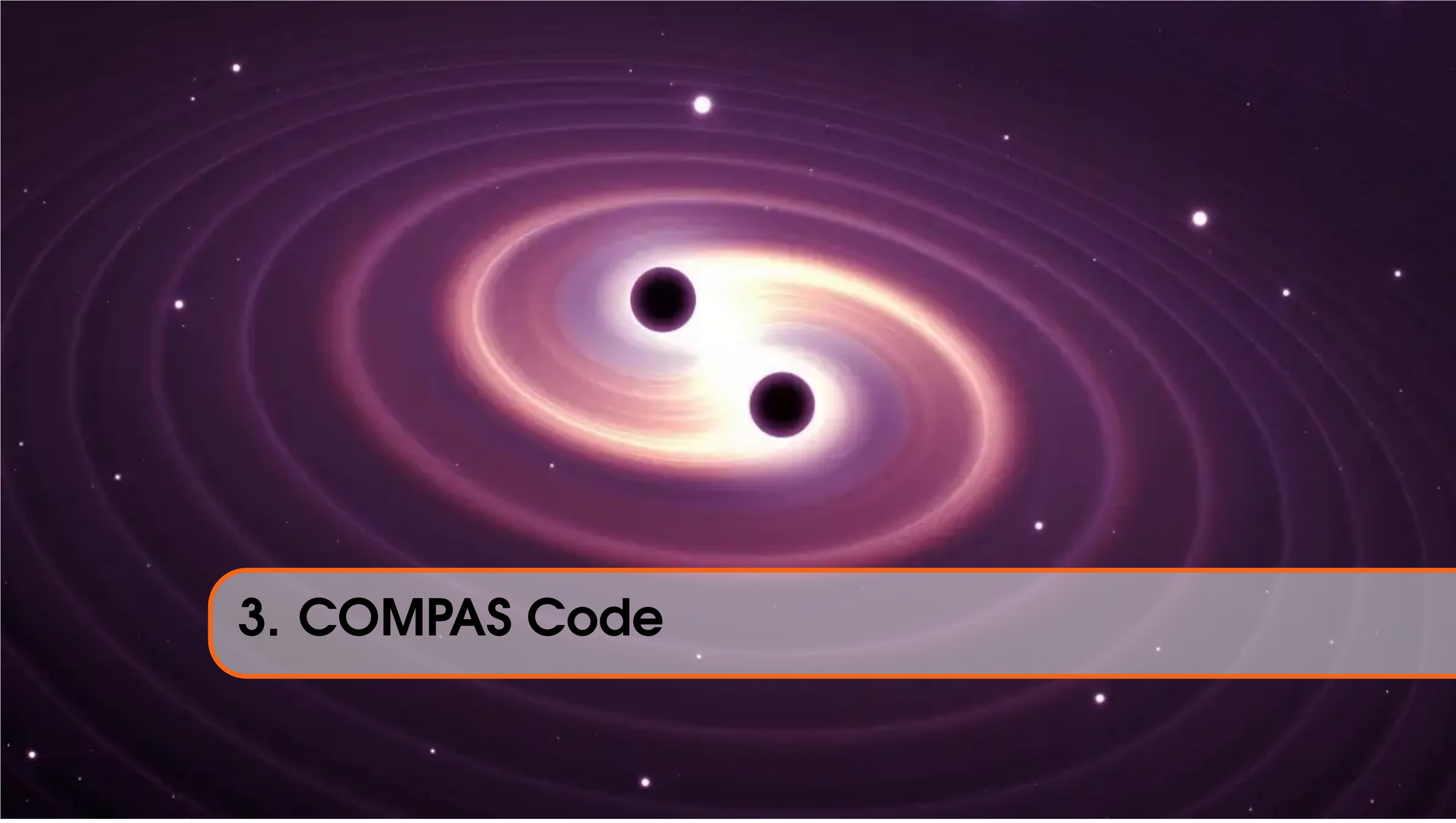
- The stars are rotating very rapidly, so fast that they efficiently transport hydrogen to the core and helium outwards, making use of the entirety of their fuel.
- At the end of the main sequence, the stars behave like Wolf-Rayet (Naked Helium) stars. So, the stars contract instead of expanding.
- As long as the metallicity of the stars is low, the wind driven mass loss is minimised, with no mass transfer as well. The binary evolves into a tight black hole binary, capable of merging within Hubble time.

### 2.4.3 Dynamical formation in dense stellar environments

- The two binary black holes may not be formed in the same binary at all
- Suppose two black holes are formed in dense stellar environments.
- A binary of a black hole and a star can be formed by a initial 3 body system, where one the object gets sufficient kinetc energy to be ejected from the system. Else, there can be a substitution case, where a black hole interrupts a stellar binary and ejects the lightest star, forming a binary with the more massive star.
- Similarly, another black hole ejects the star from the above binary, and pair of black holes is formed. If the density of the objects is high enough, the binary will be hardened until it is compact enough to merger via gravitational wave emission.

# II

# Introduction and Utilization of COMPAS



# 3. COMPAS Code

*Compact Object Mergers: Population Astrophysics Statistics*, commonly referred to as **COMPAS**, is a sophisticated and versatile rapid binary population synthesis code. It is designed to simulate the evolution of binary star systems, with a particular focus on those systems that culminate in the formation of binary black holes. The code is essential for studying the statistical properties of such systems and their potential observational signatures.

COMPAS is implemented in several C++ source files, which are compiled together to produce a single executable file. This executable can be run in multiple ways, offering flexibility to the user based on their preferred working environment or specific requirements:

- Through the terminal: Users can directly execute the COMPAS binary from the command line, allowing for efficient and straightforward simulation runs.
- Through a Python file: A Python wrapper is available, enabling users to run simulations and process outputs in a Pythonic environment. This is particularly useful for integrating COMPAS simulations with other Python-based tools and libraries.
- Through Docker: For those who prefer containerized environments, COMPAS can be run via Docker. This approach ensures consistency in the computational environment, avoiding potential issues with dependencies or software versions.

The versatility of COMPAS extends to the configurability of the simulations. Users can modify a wide range of parameters to explore different scenarios and evolutionary pathways of binary systems. These parameters influence the initial conditions, such as the masses, separation, and metallicity of the stars, as well as the physical processes that occur during the evolution, such as mass transfer, supernova kicks, and tidal interactions.

The output from COMPAS simulations is stored in the form of HDF5 files, a highly efficient and flexible data format. These files can be easily read and analyzed using various software tools. One popular approach is to process the output data in a Jupyter notebook, where users can leverage Python's rich ecosystem of scientific libraries. This setup facilitates detailed data analysis, visualization, and interpretation of the results, making it an invaluable tool for researchers in the field of astrophysics.

Moreover, COMPAS is actively developed and maintained, with regular updates and new features being added to enhance its capabilities and accuracy. The code is open-source, encouraging collaboration and contributions from the astrophysics community. This collaborative nature ensures that COMPAS remains at the cutting edge of binary population synthesis research, continually incorporating the latest theoretical developments and observational constraints.

# 4. Running Simulations

To get familiarised with the COMPAS code and the post-processing of the data, we had to complete assigned tasks which are presented as belows:

## 4.1 Task 1

In this task, we had the plot the figures for the detailed output of a single binary system. The seed for this seed was 1717786107, although this run was before I updated my COMPAS version, so cannot guarantee the same behaviour for this seed on the newer version.

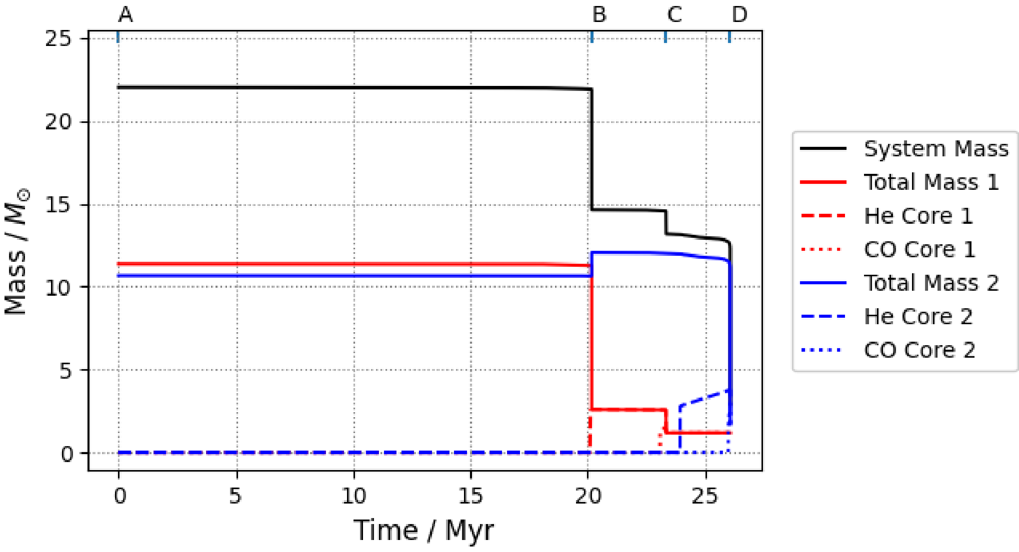


Figure 4.1: Mass($M_\odot$) vs Time(Myr)

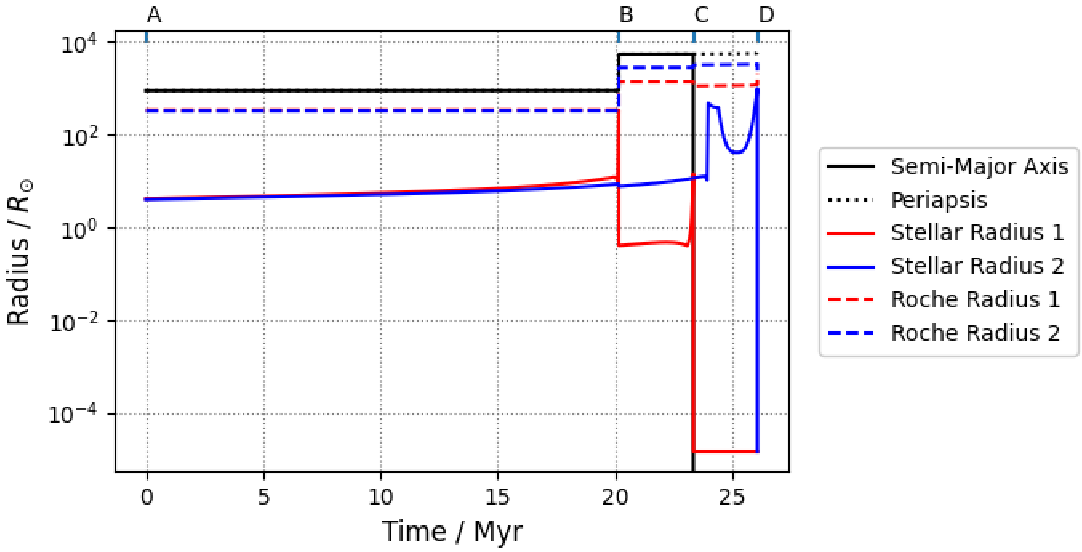


Figure 4.2: Radius($R_\odot$) vs Time(Myr)

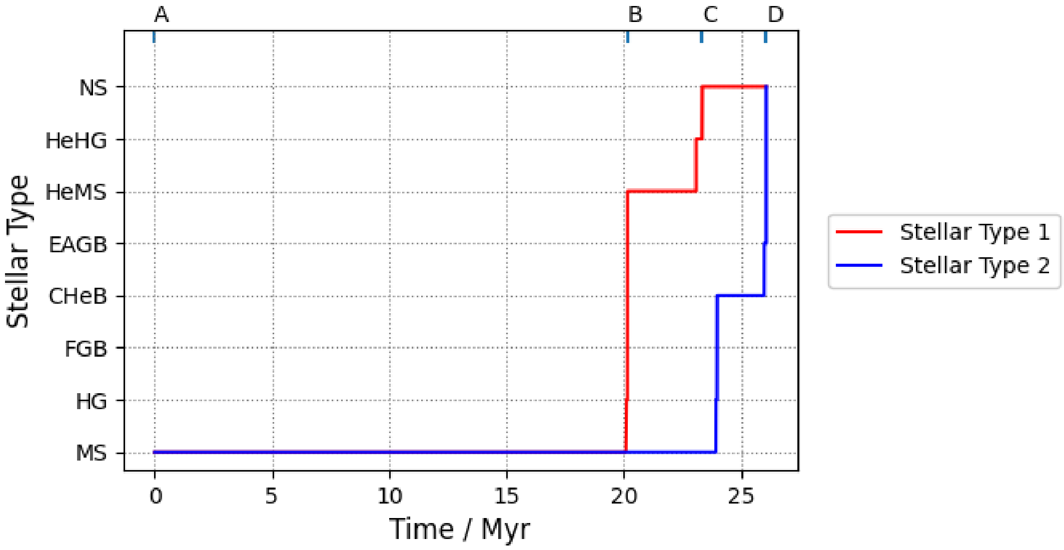


Figure 4.3: Stellar Type vs Time(Myr)

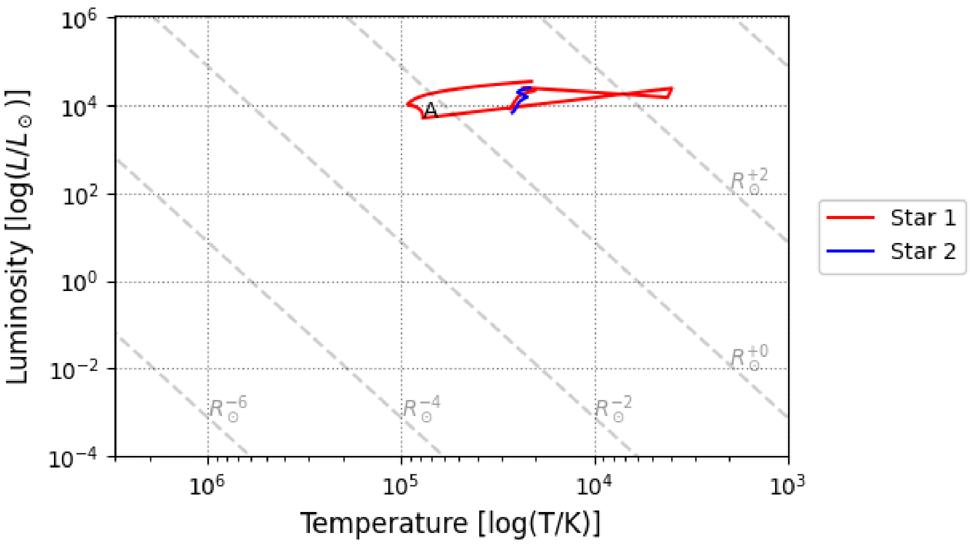
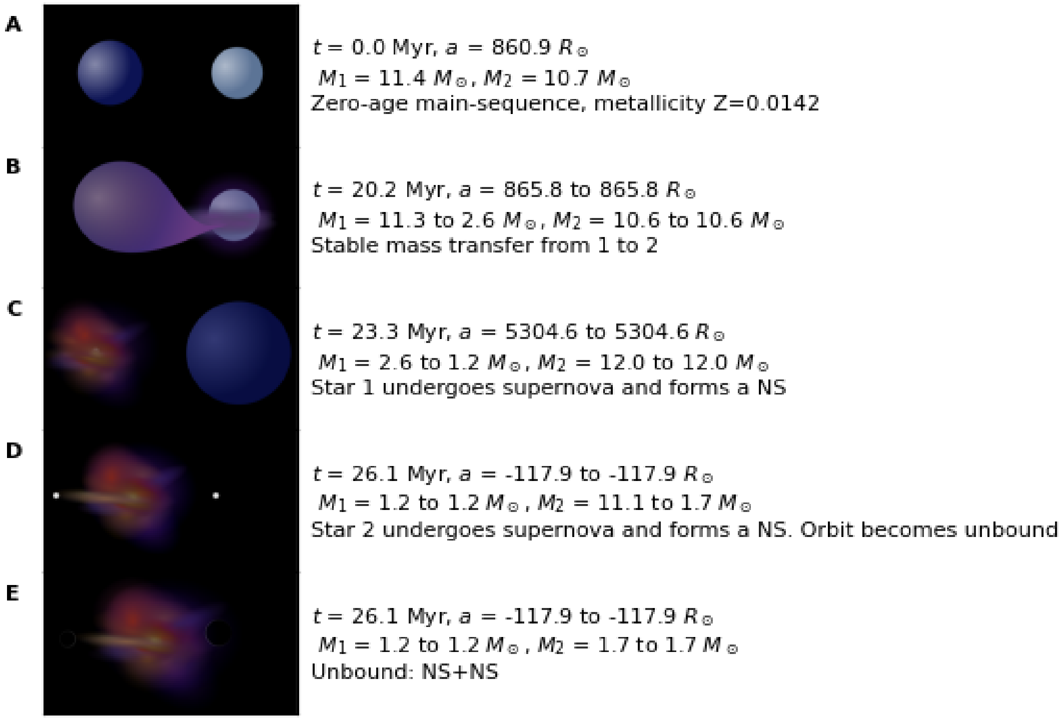


Figure 4.4: Hertzsprung-Russel Diagram



$t = 0.0$ Myr, $a = 860.9\ R_\odot$
$M_1 = 11.4\ M_\odot$, $M_2 = 10.7\ M_\odot$
Zero-age main-sequence, metallicity Z=0.0142

$t = 20.2$ Myr, $a = 865.8$ to $865.8\ R_\odot$
$M_1 = 11.3$ to $2.6\ M_\odot$, $M_2 = 10.6$ to $10.6\ M_\odot$
Stable mass transfer from 1 to 2

$t = 23.3$ Myr, $a = 5304.6$ to $5304.6\ R_\odot$
$M_1 = 2.6$ to $1.2\ M_\odot$, $M_2 = 12.0$ to $12.0\ M_\odot$
Star 1 undergoes supernova and forms a NS

$t = 26.1$ Myr, $a = -117.9$ to $-117.9\ R_\odot$
$M_1 = 1.2$ to $1.2\ M_\odot$, $M_2 = 11.1$ to $1.7\ M_\odot$
Star 2 undergoes supernova and forms a NS. Orbit becomes unbound

$t = 26.1$ Myr, $a = -117.9$ to $-117.9\ R_\odot$
$M_1 = 1.2$ to $1.2\ M_\odot$, $M_2 = 1.7$ to $1.7\ M_\odot$
Unbound: NS+NS

Figure 4.5: Timeline of events with visual illustrations

- In this seed, the system evolves to form an unbound NS+NS sytem.
- At t = 20.2 Myr, star 1 starts transferring its mass to star 2, stably. However since the process is not 100% efficient, star 2 only gains a fraction of the donated mass.
- After this, at t = 23.3 Myr, star 1 undergoes supernova to form a neutron star, leaving the system intact.

- Then, at t = 26.1 Myr, star 2 undergoes supernova to form a neutron star. However, this supernova has a high SN kick associated with it. The neutron star is ejected from the system and the binary becomes unbound.
- Figure 3.4 shows the evolution of both the stars on HR diagram, until star 1 undergoes supernova. We can see that star 1 evolves a lot more than star 2 on the plot.

## 4.2 Task 2

In this task, we had to run 10,000 runs and count the number of binaries that interacted through different mass transfer channels. Also, we had to count the distribution of occurences of supernovae among these binaries. These 10,000 binaries were simulated on default settings.
The first half of the task was counting the number of different number of events that occur in the 10,000 binaries. The overall statistics for this simulation have been summarised in the table below:

| Statistic | Value |
|---|---|
| Time taken to simulate 10,000 binaries | 143.928 seconds |
| Fraction of binaries that never interact | 0.4104 |
| Fraction of binaries that experience only unstable mass transfer | 0.1354 |
| Fraction of binaries that experience only stable mass transfer | 0.1313 |
| Fraction of binaries that undergo stellar merger | 0.3229 |
| Total number of supernovae | 6861 |
| Fraction of binaries with 0 supernovae | 0.5324 |
| Fraction of binaries with 1 supernovae | 0.2491 |
| Fraction of binaries with 2 supernovae | 0.2185 |

Table 4.1: Simulation statistics for 10,000 binaries

Following this, we had to plot the binaries against their initial parameters and see if there are any observable trends.

### 4.2.1 Plots

The most intuitive initial parameters that one can think are MASS@ZAMS(1), MASS@ZAMS(2) and SemiMajorAxis@ZAMS. So, to begin with the graphs, I plotted these three parameters in 3D scatter plot to get a rough idea of the distribution.

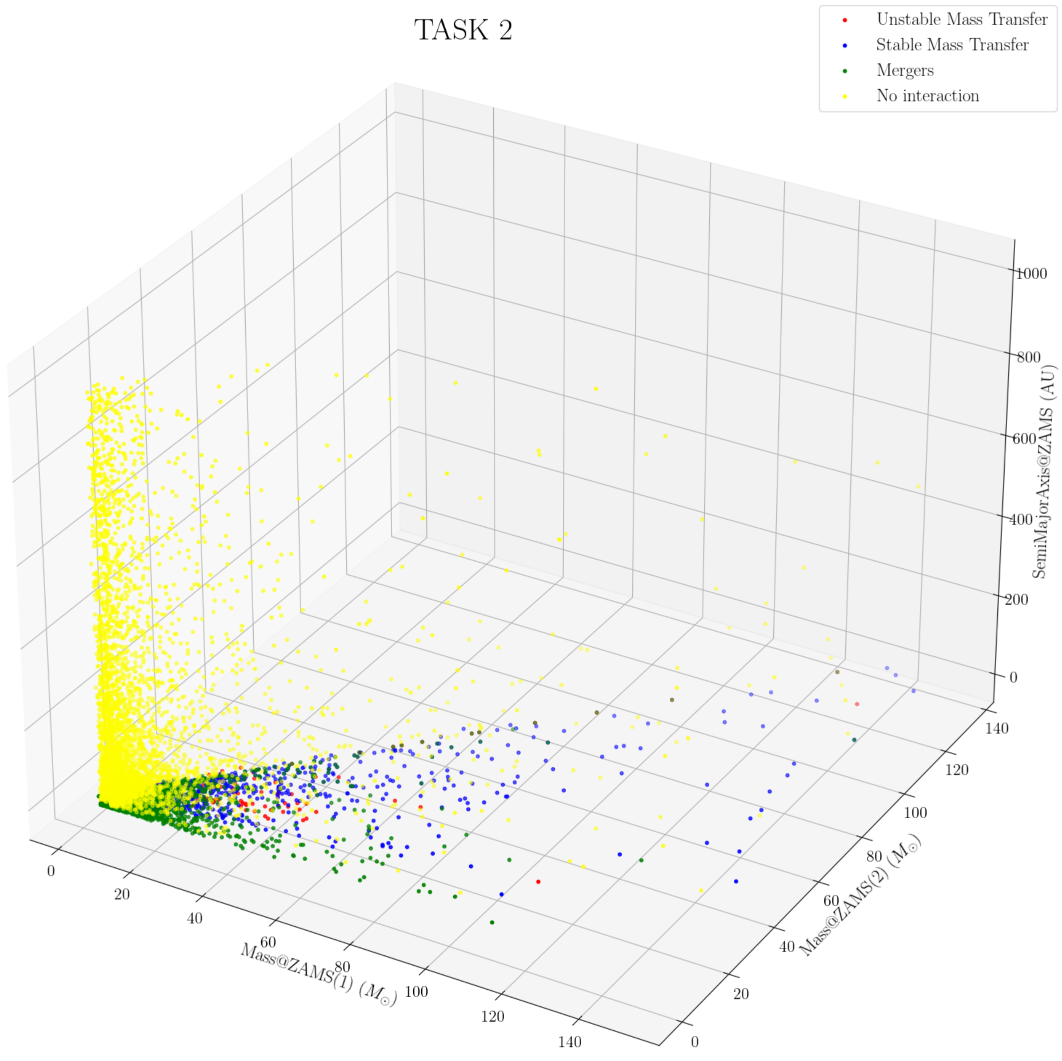


Figure 4.6: 3D Scatter Plot of Initial Parameters

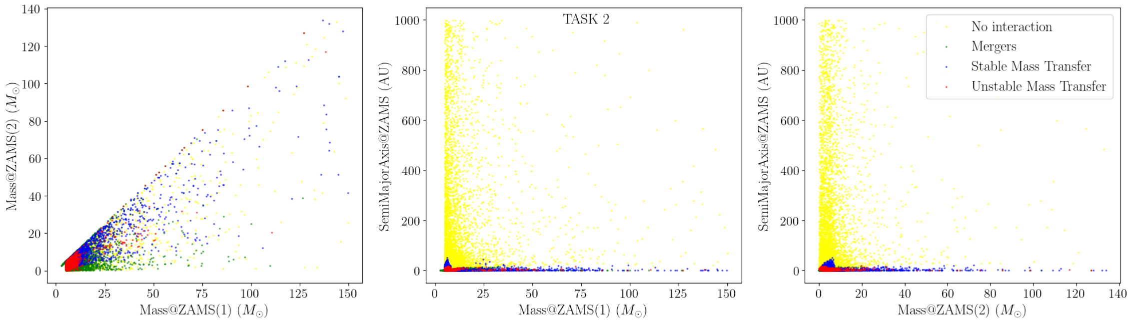


Figure 4.7: Projections of 3D scatter plot

We can see that higher values of SemiMajorAxis@ZAMS are dominated by the binaries having no interaction between the stars. By setting the limit of this axis to 20 AU, we can start to see the other types of the interactions in the binaries.

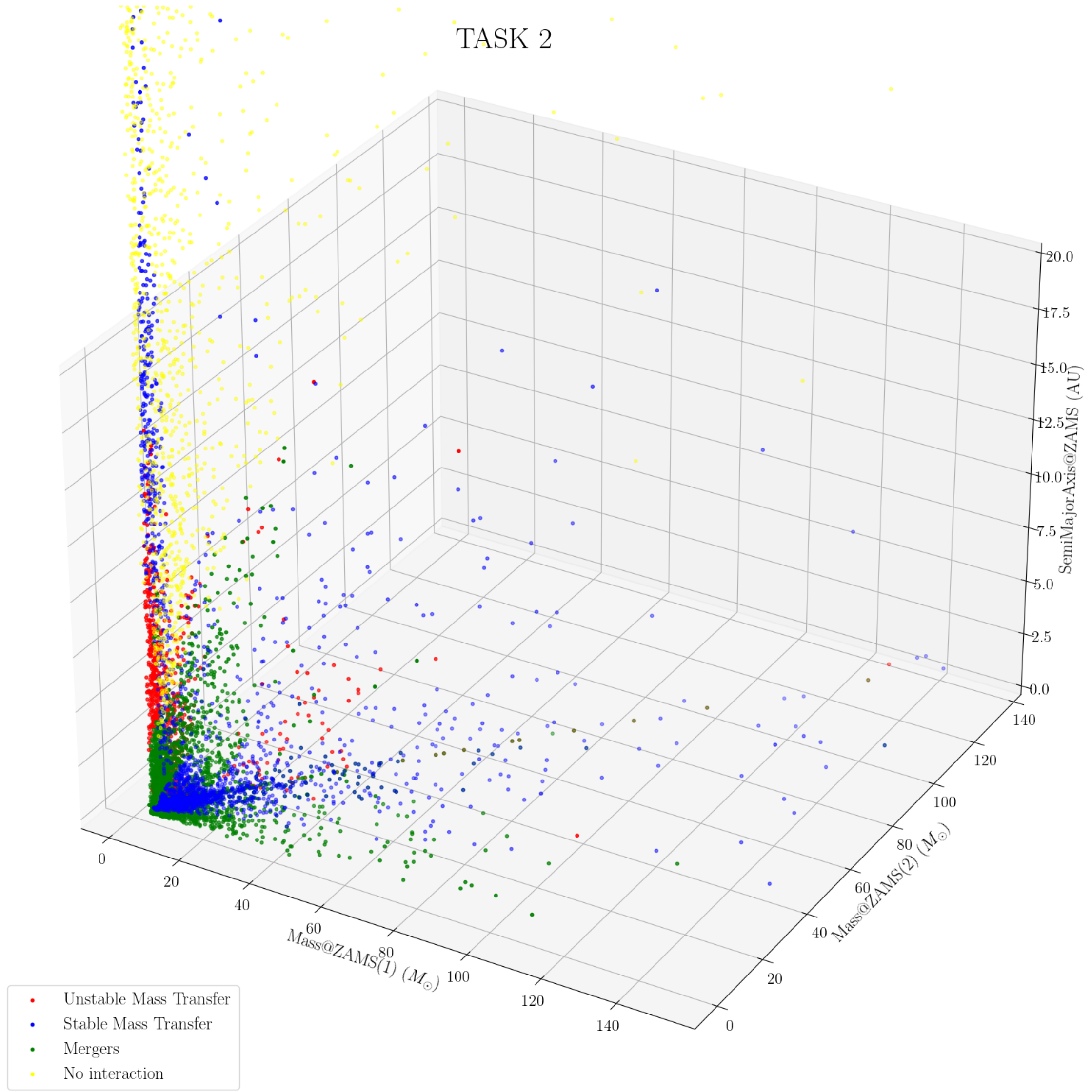


Figure 4.8: 3D Scatter Plot of Initial Parameters, with upper limit of SemiMajorAxis@ZAMS set to 20 AU

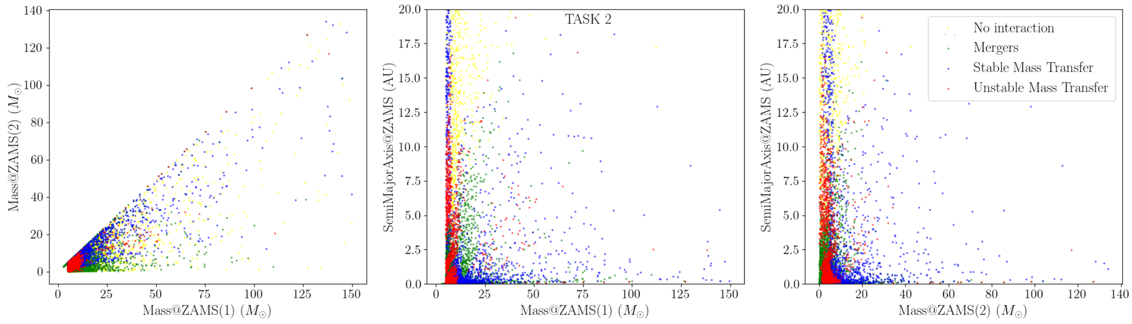


Figure 4.9: Projections of 3D scatter plot, with upper limit of SemiMajorAxis@ZAMS set to 20 AU

From the three projections of this scatter plot, the plot of MASS@ZAMS(1) vs SemiMajorAxis@ZAMS appears the most promising to look for trends between the different types of interactions.

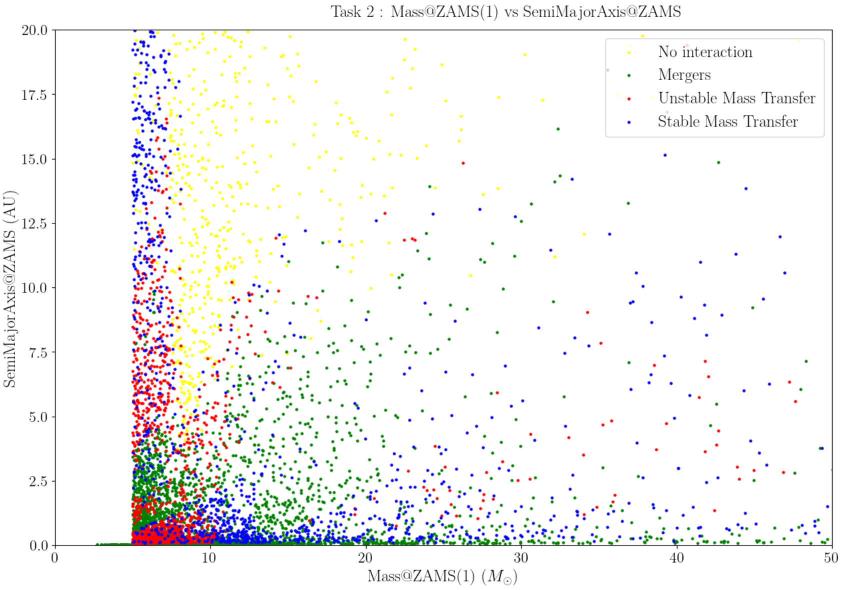


Figure 4.10: MASS@ZAMS(1) vs SemiMajorAxis@ZAMS

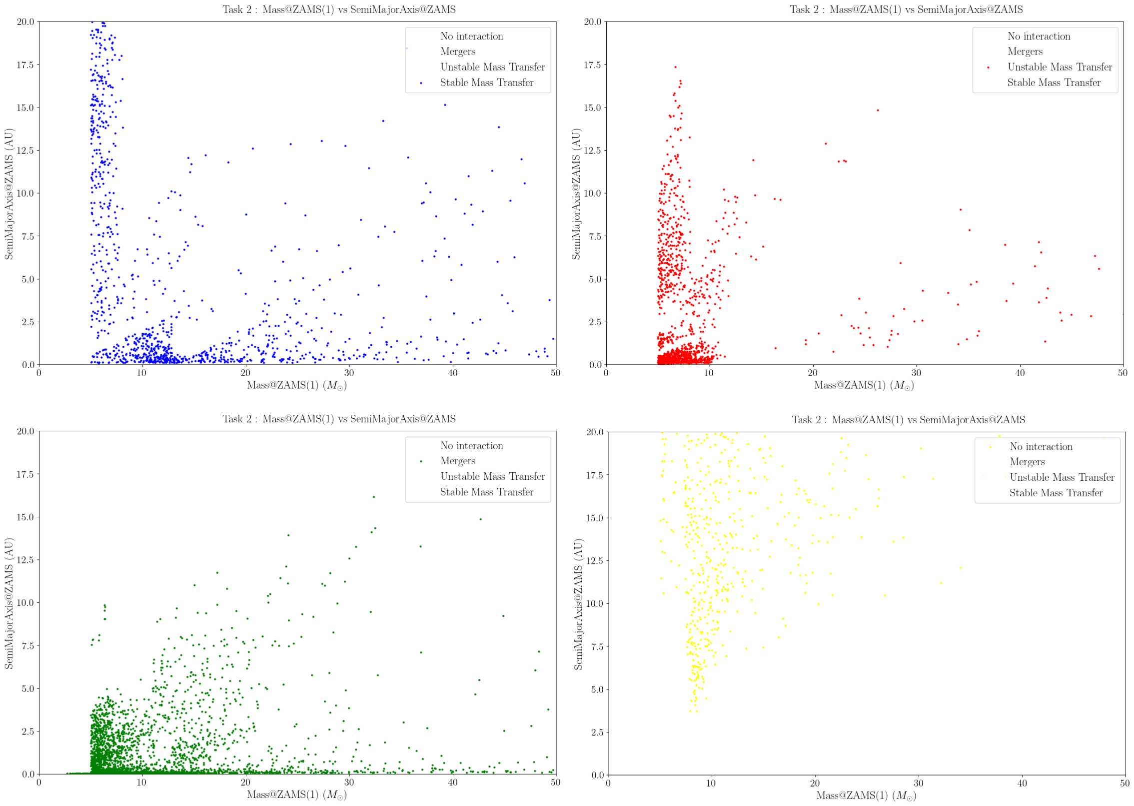


Figure 4.11: Different types of interaction channels

The above figures have SemiMajorAxis@ZAMS limited to 20 AU, since for greater values, the dominant channel is that of no-interaction between the stars. We can see the full picture by plotting the Y-axis on a logscale as below.

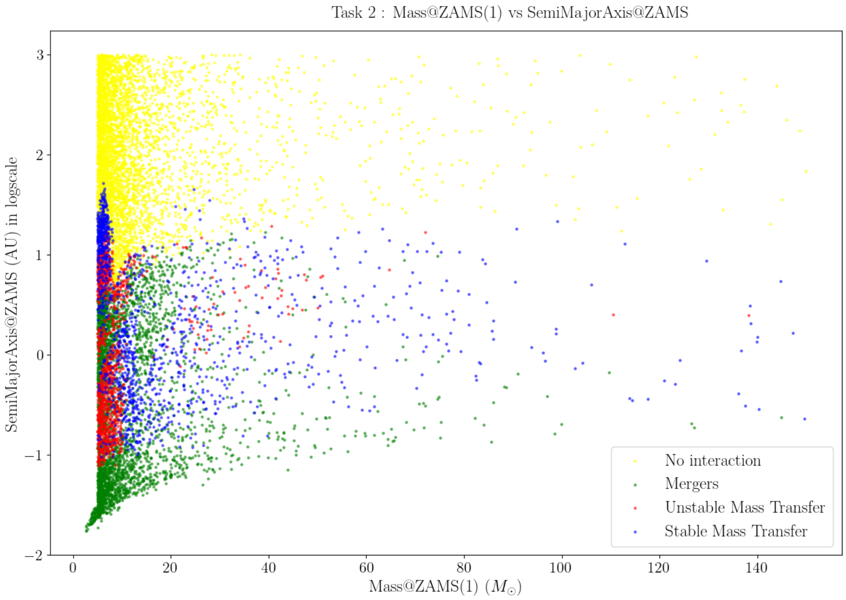


Figure 4.12: Mass@ZAMS(1) vs SemiMajorAxis@ZAMS(as powers of 10)

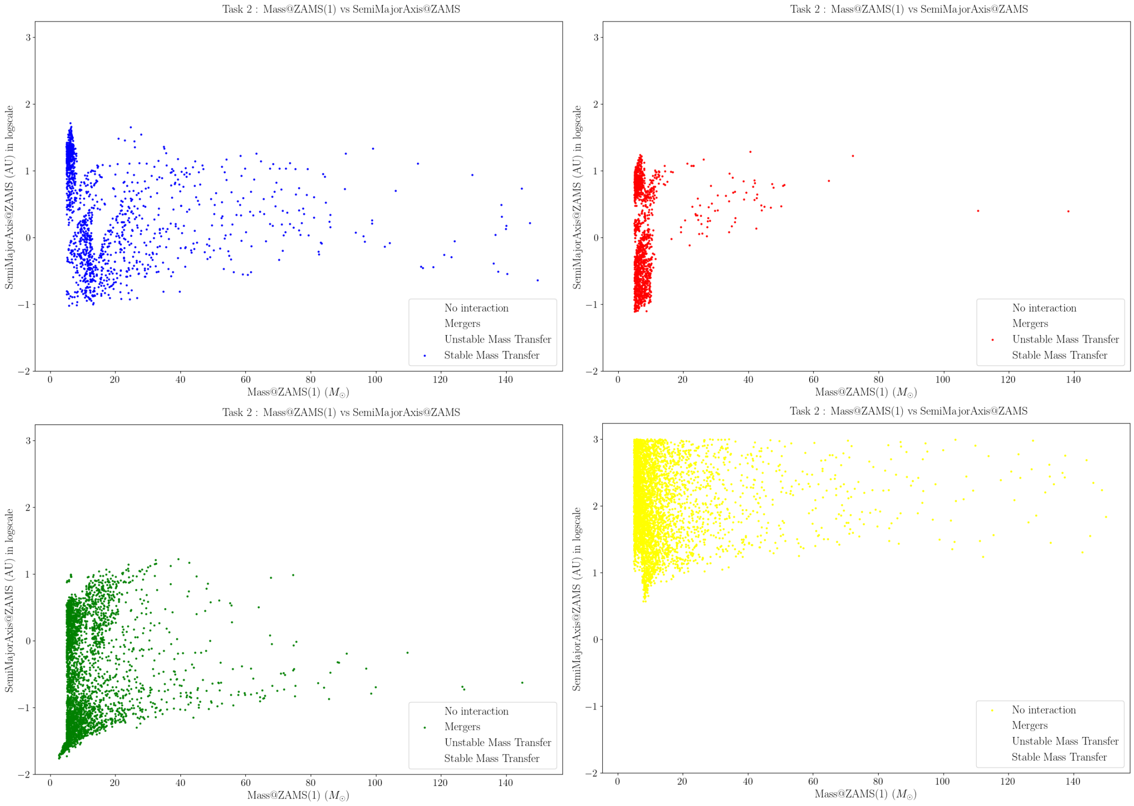


Figure 4.13: Different types of interaction channels

### 4.2.2 Observations

1. Stable Mass Transfers:
    - There are a total of 1313 binaries that experience only stable mass transfer.
    - Majority of stable mass transfers occur only when the SemiMajorAxis@ZAMS is less than 20 AU. However, the systems having primary mass (@ZAMS) around 5-8 $M_\odot$ are able to experience stable mass transfer even when the semi-major axis (@ZAMS) is as high as 40 AU.
    - The primary masses of these systems vary from 5 $M_\odot$ to 150 $M_\odot$, while the secondary masses vary from 0.1 (minimum limit) all the way to 140 $M_\odot$.
2. Unstable Mass Transfer (Common Envelope Evolution)
    - There are a total of 1354 binaries that experience unstable mass transfer.
    - Most of these systems of have semi-major axis less than 10 AU, with only a few experiencing Common Envelope Evolution near 15 AU.
    - A large majority of these systems have masses of the stars to be less than 10 $M_\odot$, with only a few heavier systems experiencing CEE uptill the limit of 50 $M_\odot$.
3. Mergers:
    - There are total of 3229 systems that undergo stellar mergers.
    - Almost all mergers take place when the semi major axis is less than 10 AU.
    - The masses of the stars can vary from 0.1 $M_\odot$ to 100 $M_\odot$, with only a handful systems having masses of the stars beyond 100.
    - One interesting thing to notice is that in all the other interactions, except mergers, the mass of the primary star seems have minimum value of around 5 $M_\odot$.
    - However, when the stars in the systems are really close (semi-major axis from 0.01 to 0.1 AU), the mass of the primary can be less than 5 $M_\odot$, however these sytems always result in mergers of the two stars. This is indicated by the little tip protruding out from the scatter plot of primary mass vs log of semi-major axis, for mergers(bottom left), in fig 4.13 .
4. No interaction:
    - The total number of systems that do not interact are 4104.
    - Almost all of these systems have semi-major axis to be greater than $\approx 10$ AU, all the way upto 1000 AU.
    - The masses of the stars can vary from 0-5 AU to 140 AU.
    - One interesting observation is that systems with primary mass around 10 AU do not seem to interact even when the semi-major axis is as low as $\approx 5$ AU. This can be seen from the downward tip of the graph of Mass@ZAMS(1) vs log of SemiMajorAxis@ZAMS in fig 4.13 (bottom right), or from the very prominent dip of the scatter plot of Mass@ZAMS vs SemiMajorAxis@ZAMS in fig 4.11 (bottom right).

## 4.3 Task 3

In this task, we had to run 10,000 binaries with conservative mass transfer. To accomplish this, the Mass Transfer Accretion Efficiency was fixed to 1. All the other parameters were set to default.

The statistics for the task are as follows:

| Statistic | Value |
|---|---|
| Time taken to simulate 10,000 binaries | 99.6698 seconds |
| Fraction of binaries that never interact | 0.3990 |
| Fraction of binaries that experience only unstable mass transfer | 0.1279 |
| Fraction of binaries that experience only stable mass transfer | 0.1076 |
| Fraction of binaries that undergo stellar merger | 0.3655 |
| Total number of supernovae | 6995 |
| Fraction of binaries with 0 supernovae | 0.5165 |
| Fraction of binaries with 1 supernovae | 0.2675 |
| Fraction of binaries with 2 supernovae | 0.2160 |

Table 4.2: Simulation statistics for 10,000 binaries

### 4.3.1 Plots

The plots for Task-3 are also in the same order, beginning with a 3D scatter plot of the three initial parameters (Mass@ZAMS(1), Mass@ZAMS(2) and SemiMajorAxis@ZAMS), followed by scatter plots of Mass@ZAMS(1) vs SemiMajorAxis@ZAMS.

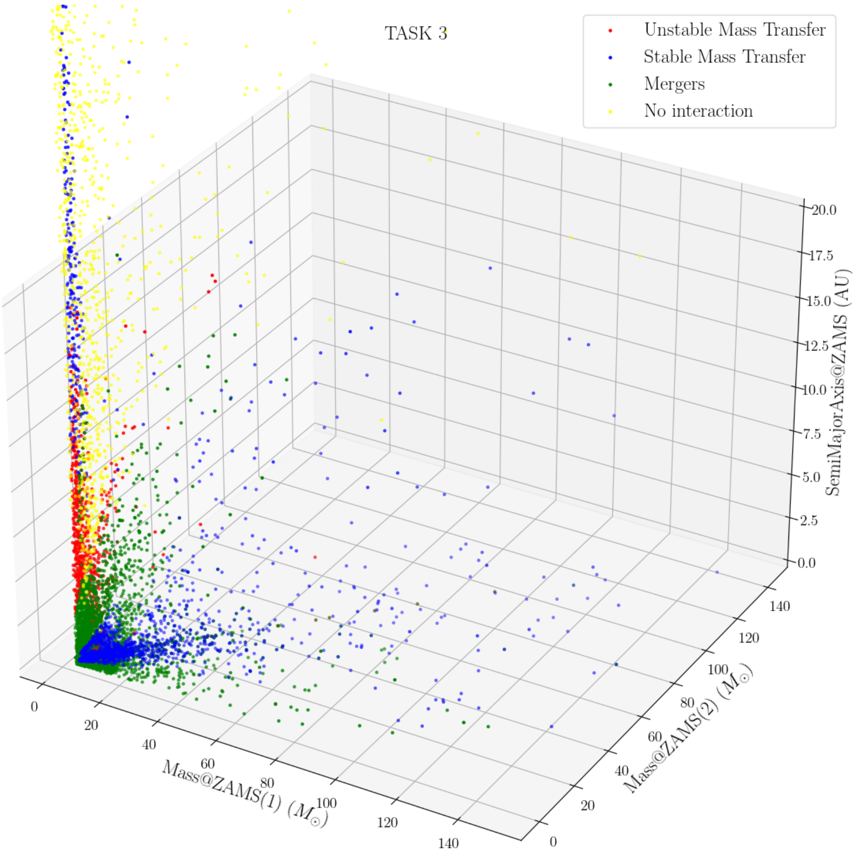


Figure 4.14: 3D Scatter Plot of Initial Parameters, with upper limit of SemiMajorAxis@ZAMS set to 20 AU

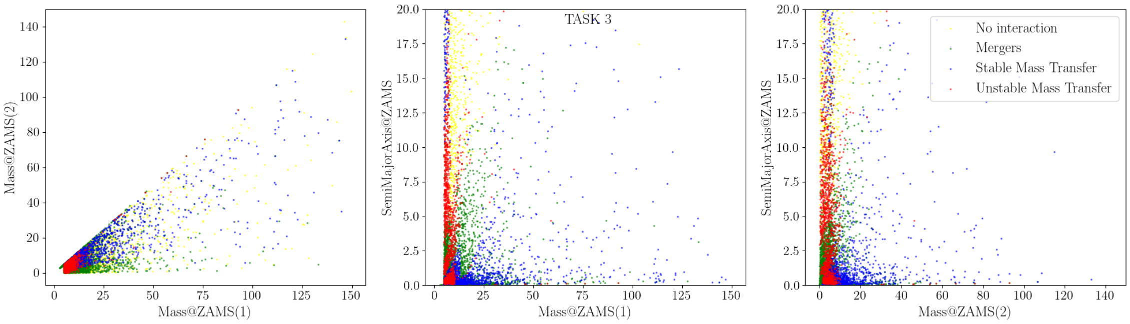


Figure 4.15: Projections of 3D scatter plot, with upper limit of SemiMajorAxis@ZAMS set to 20 AU

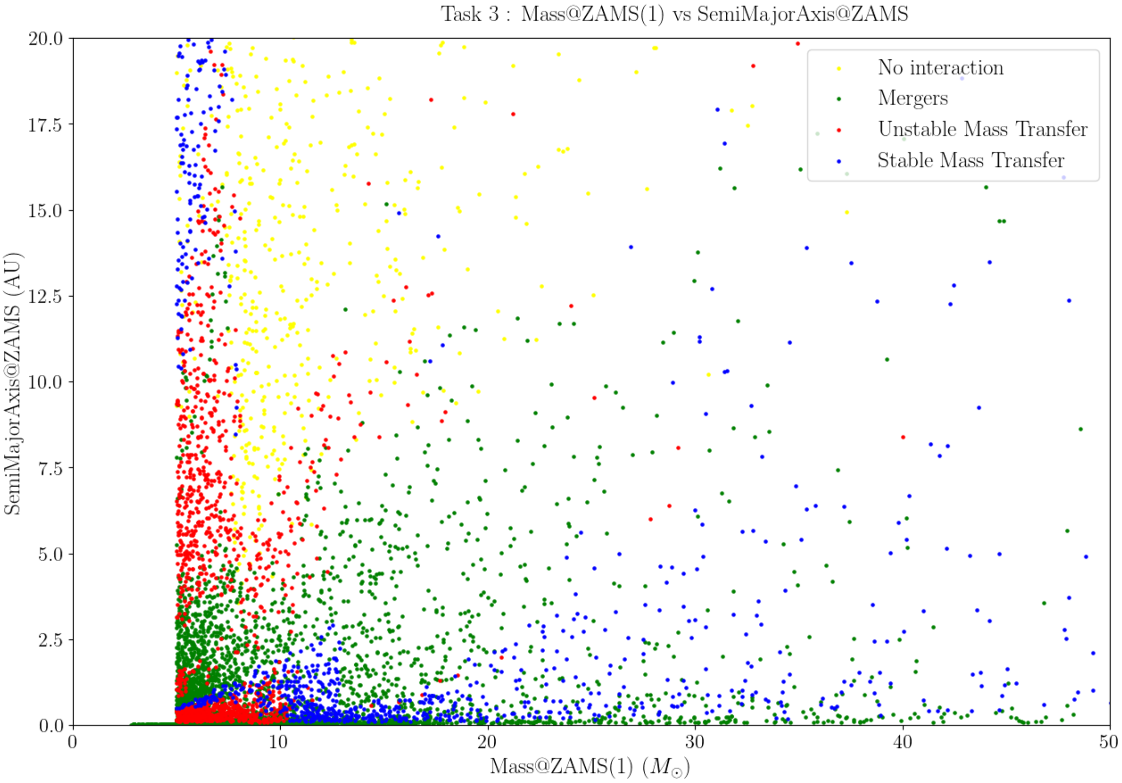


Figure 4.16: Mass@ZAMS(1) vs SemiMajorAxis@ZAMS

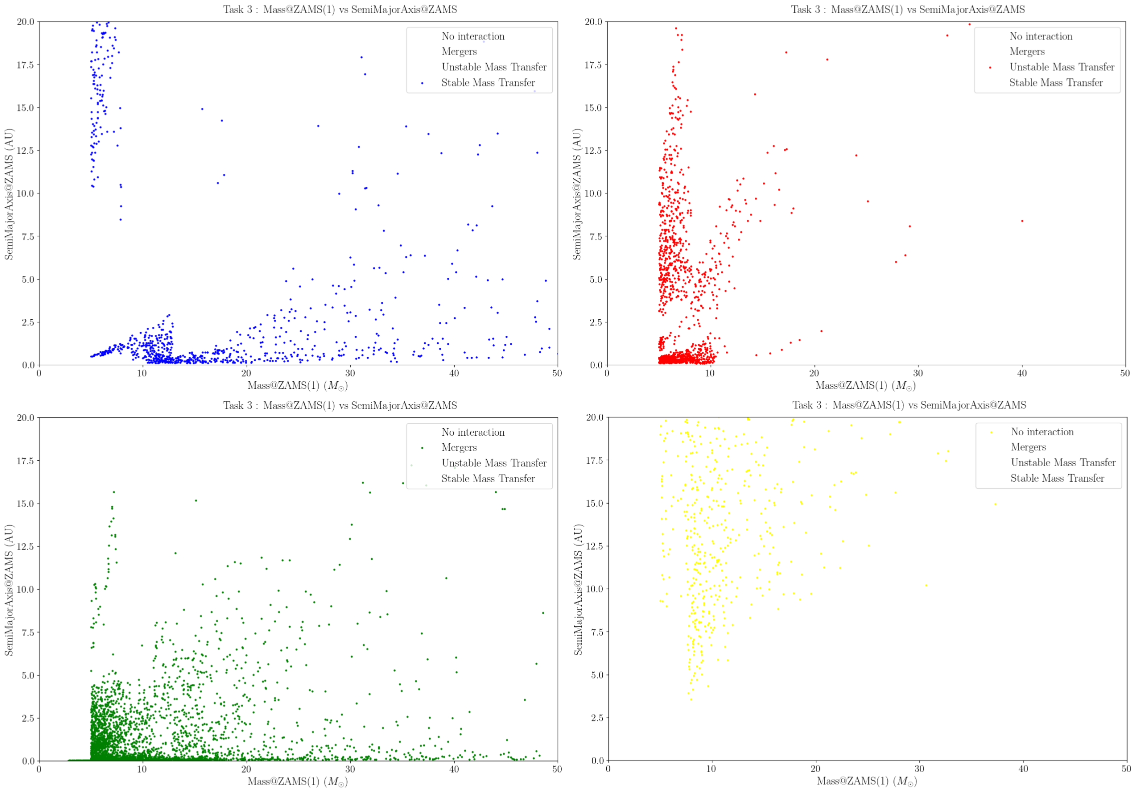


Figure 4.17: Different types of interactions

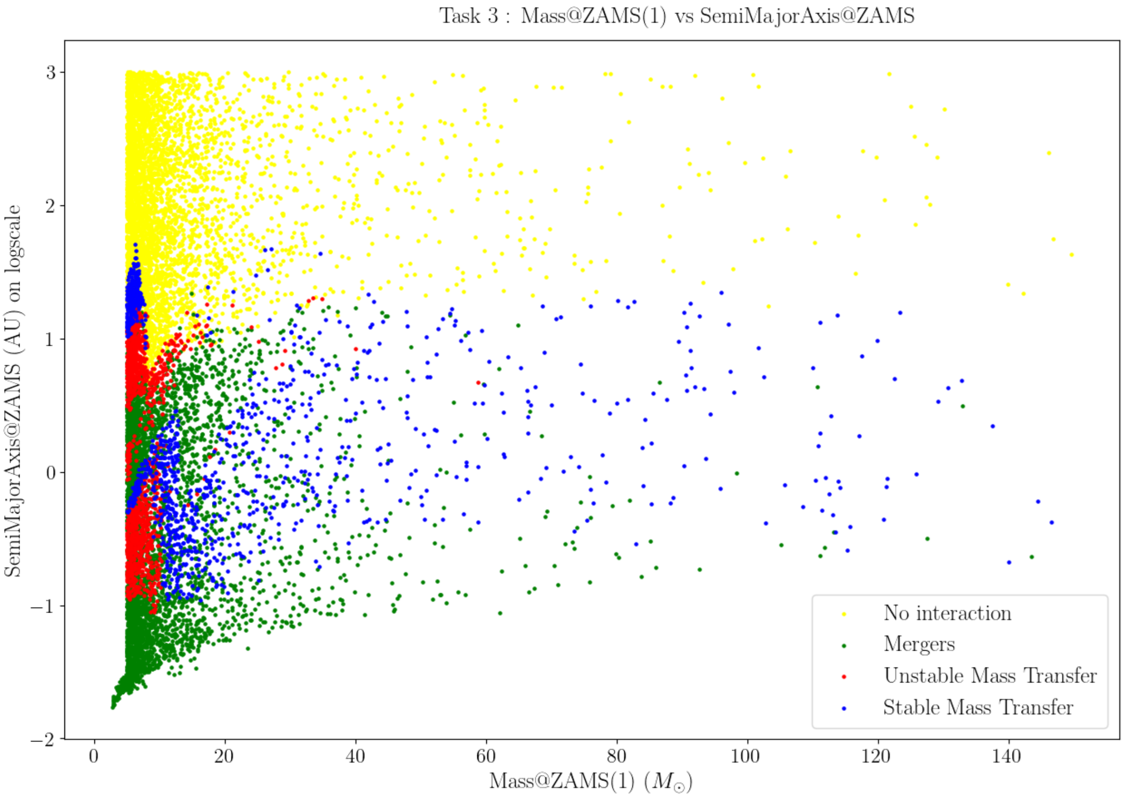


Figure 4.18: Mass@ZAMS vs log of SemiMajorAxis@ZAMS

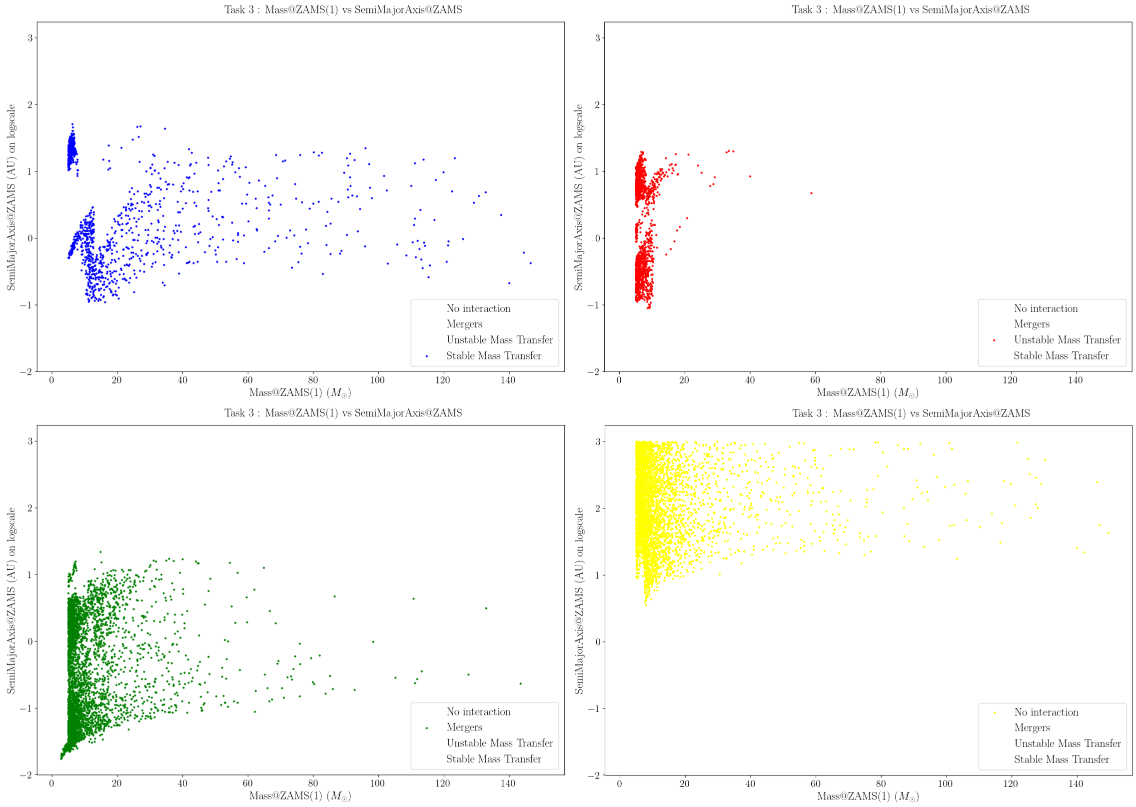


Figure 4.19: Different types of interactions

### 4.3.2 Observations

1. Stable Mass Transfers:
    - A total of 1076 binaries undergo only stable mass transfer.
    - These binaries have semi-major axis ranging from 0.1 AU to around 50 AU.
    - The primary mass ranges from 5 $M_\odot$ to 140 $M_\odot$ while the secondary mass ranges from 0.1 $M_\odot$ to 140 $M_\odot$.
2. Unstable Mass Transfer (Common Envelope Evolution):
    - The number of binaries that develop common envelope are 1279.
    - The semi-major axis lies between 0.1 AU and 20 AU.
    - Almost all systems have masses of stars less than 20 $M_\odot$, with only a few having mass ranging from 40 to 60 $M_\odot$.
3. Mergers:
    - 3655 binaries undergo stellar mergers.
    - The upper limit of the semi-major axis in these systems is around 10 AU.
    - The stellar masses can vary from minimum of a few solar masses all the way upto 140 $M_\odot$, however only a few systems have masses over 100 $M_\odot$.
    - At extremely low values of semi-major axis, the primary mass again goes below 5 $M_\odot$. These systems also always result in a merger.
4. No interaction:
    - The number of binaries that do not interact is 3990.
    - Almost all of these have semi-major axis greater than 10 AU, and go uptill 1000 AU.
    - The masses of stars also range from a few solar masses uptill around 140 $M_\odot$.

## 4.4 Differences between Task 2 and Task 3

- Task 2 was run using all default parameters, while Task 3 was run such that mass transfer between the stars was fully conservative. This means that during stable mass transfer, all the mass lost by the donor star was fully accreted on the other.
- My hypothesis was that this would result in more systems undergoing stable mass transfer, however the observation was completely opposite.
- The number systems undergoing stable mass transfer actually drops from 1313 to 1076.

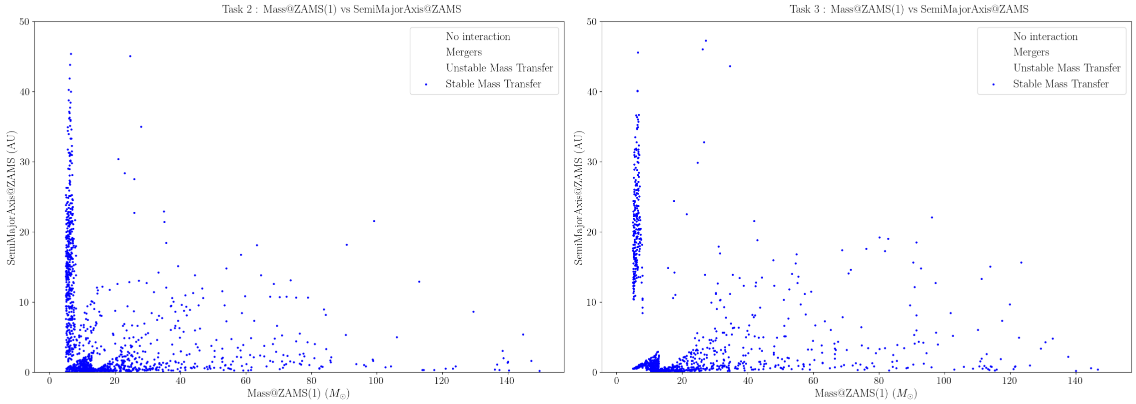


Figure 4.20: Difference in the distribution of stable mass transfer in Task 2 and Task 3

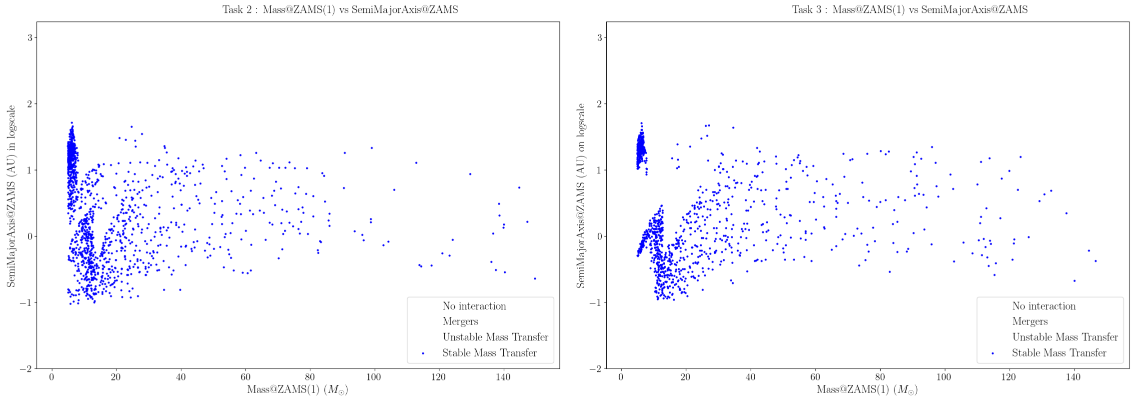


Figure 4.21: Difference in distribution of stable mass transfer in Task 2 and Task 3, with Y-axis on logscale

- We can see this absence the figures above. The systems with primary masses $\approx 5M_\odot$ and semi major axis less than 10 AU are absent in the Task 3.
- One possible explanation is that these sytems merge instead of/along with having other types of mass transfer, as can be inferenced from the increased in number of mergers from 3229 to 3655.

## 4.5 Task 4

In this task, we simulated 100,000 binaries on default settings. This task was focused on the Double Compact Objects (DCOs). These DCOs include binary black holes, binary neutron stars and black hole + neutron star systems. The statistics of this task are as given below:

| **Statistic** | **Value** |
|---|---|
| Time taken to simulate 100,000 binaries | 331.023 seconds |
| Total number of DCOs formed | 4571 |
| Number of DCOs merging in Hubble time | 495 |
| Number of binary black holes | 4283 |
| Number of binary black holes merging in Hubble Time | 343 |
| Number of black hole + neutron star | 246 |
| Number of black hole + neutron star merging in Hubble Time | 120 |
| Number of binary neutron stars | 42 |
| Number of binary neutron stars merging in Hubble Time | 32 |

Table 4.3: Statistics for DCOs obtained from simulating 100,000 binaries

After this, we had to plot histograms of primary mass, secondary mass, mass ratio and chirp mass for these DCOs.

What is **Chirp Mass**?

Chirp mass in DCOs is a quantity composed of the component masses as follows

$$\mu = \frac{(m_1 \times m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

It gravitational wave data analysis, chirp mass can be measured easily and more accurately than the component masses of the binary.

### 4.5.1 Binary black Holes

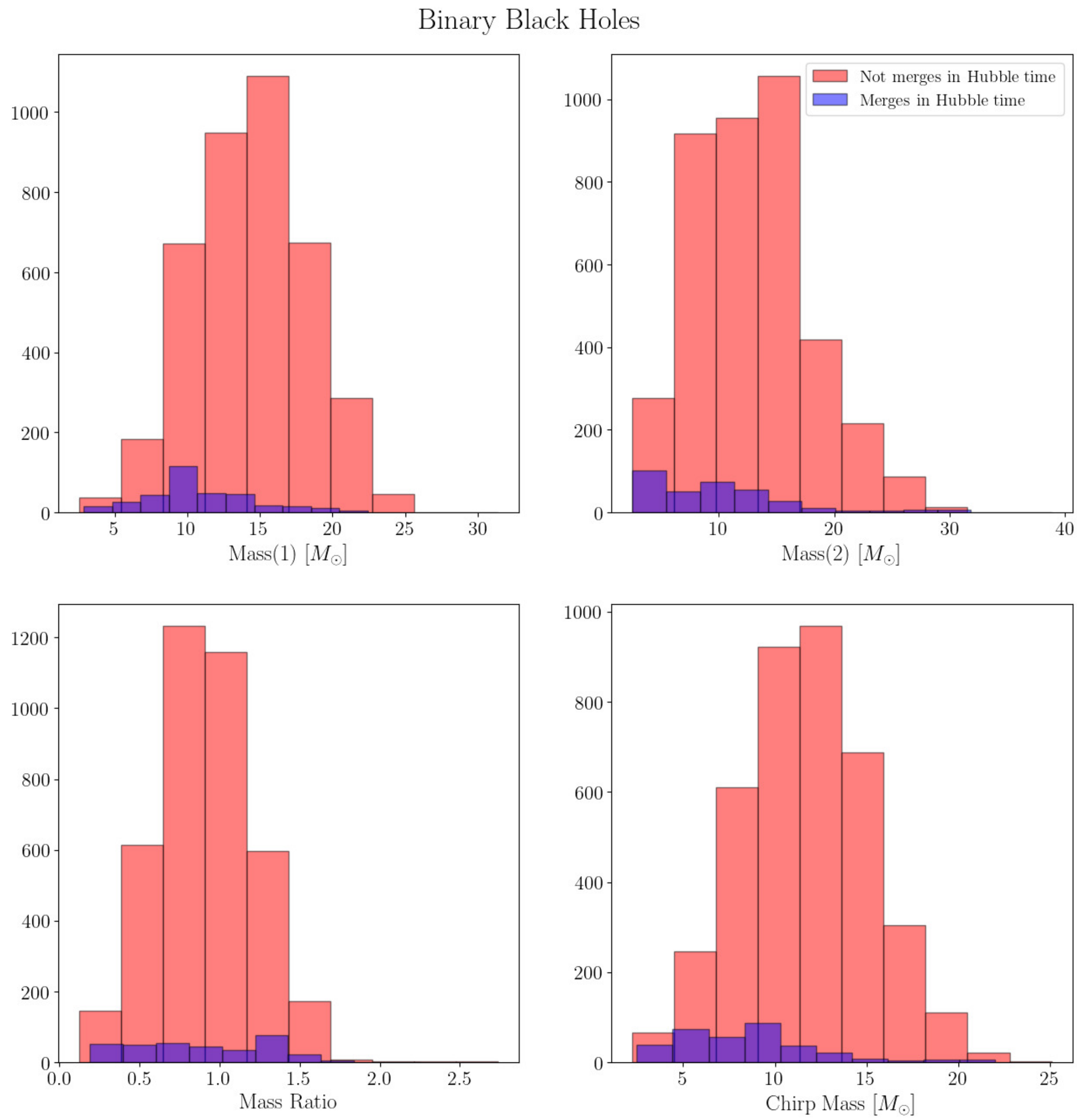


Figure 4.22: Histograms for Binary Black Holes

- Binary black holes are by far the most common types of DCOs formed. A total 4283 binary black holes were formed, out of which only 343 merged in Hubble time.
- For the chirp mass distribution, the peak is in the range around 12-14 $M_\odot$. However, for the systems merging in Hubble time, the peak is around 10 $M_\odot$.

### 4.5.2 Black Hole + Neutron Star

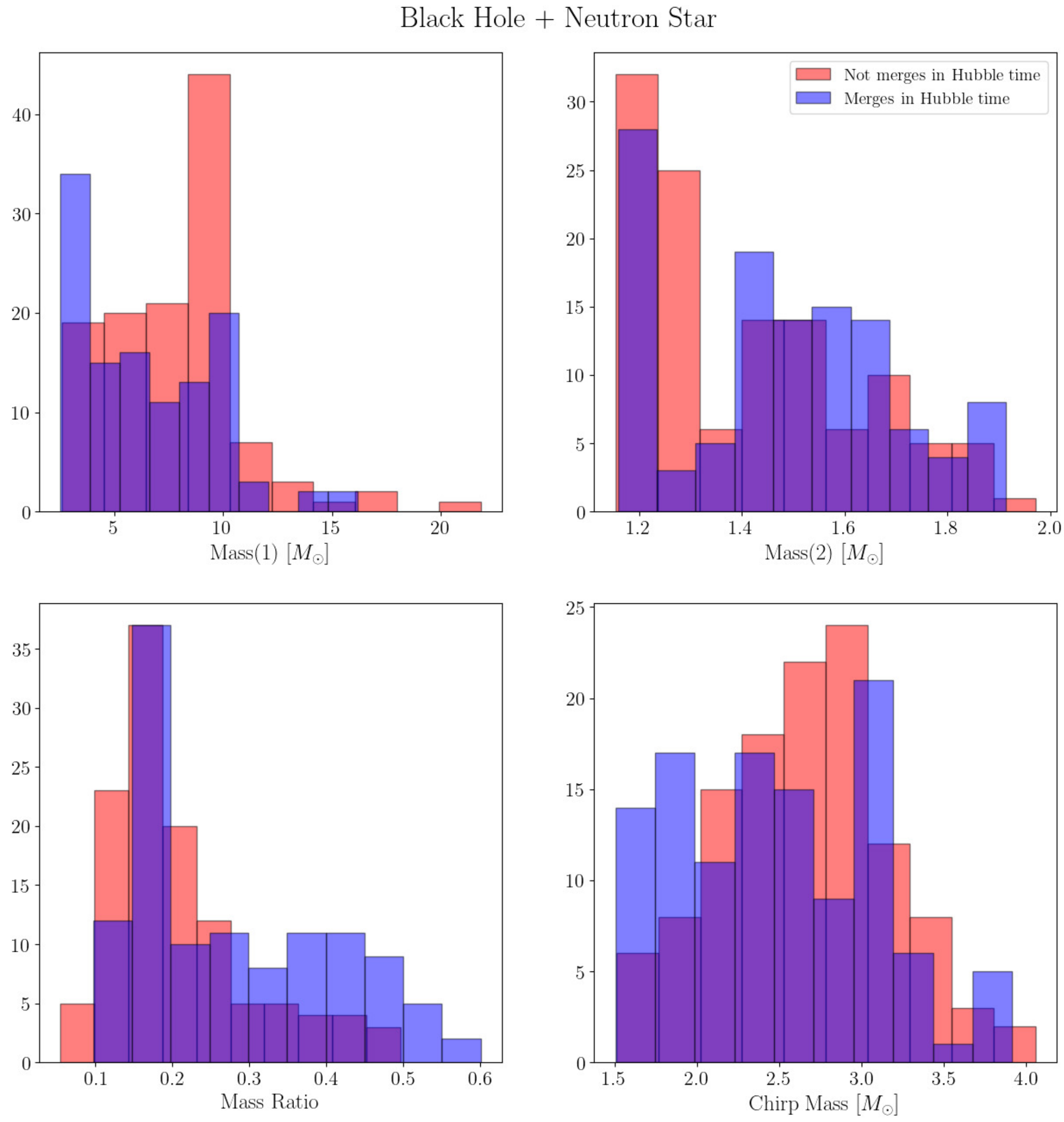


Figure 4.23: Histograms for Black Hole + Neutron Star

- Black hole + Neutron star systems are fewer in number, with only 246 forming in 100,000. Half of these merge within hubble time.
- The chirp mass distribution of these DCOs peak at around 3 $M_\odot$. This is because the neutron stars are not that massive, having masses in the range from 1-2 $M_\odot$.

### 4.5.3 Binary Neutron Stars

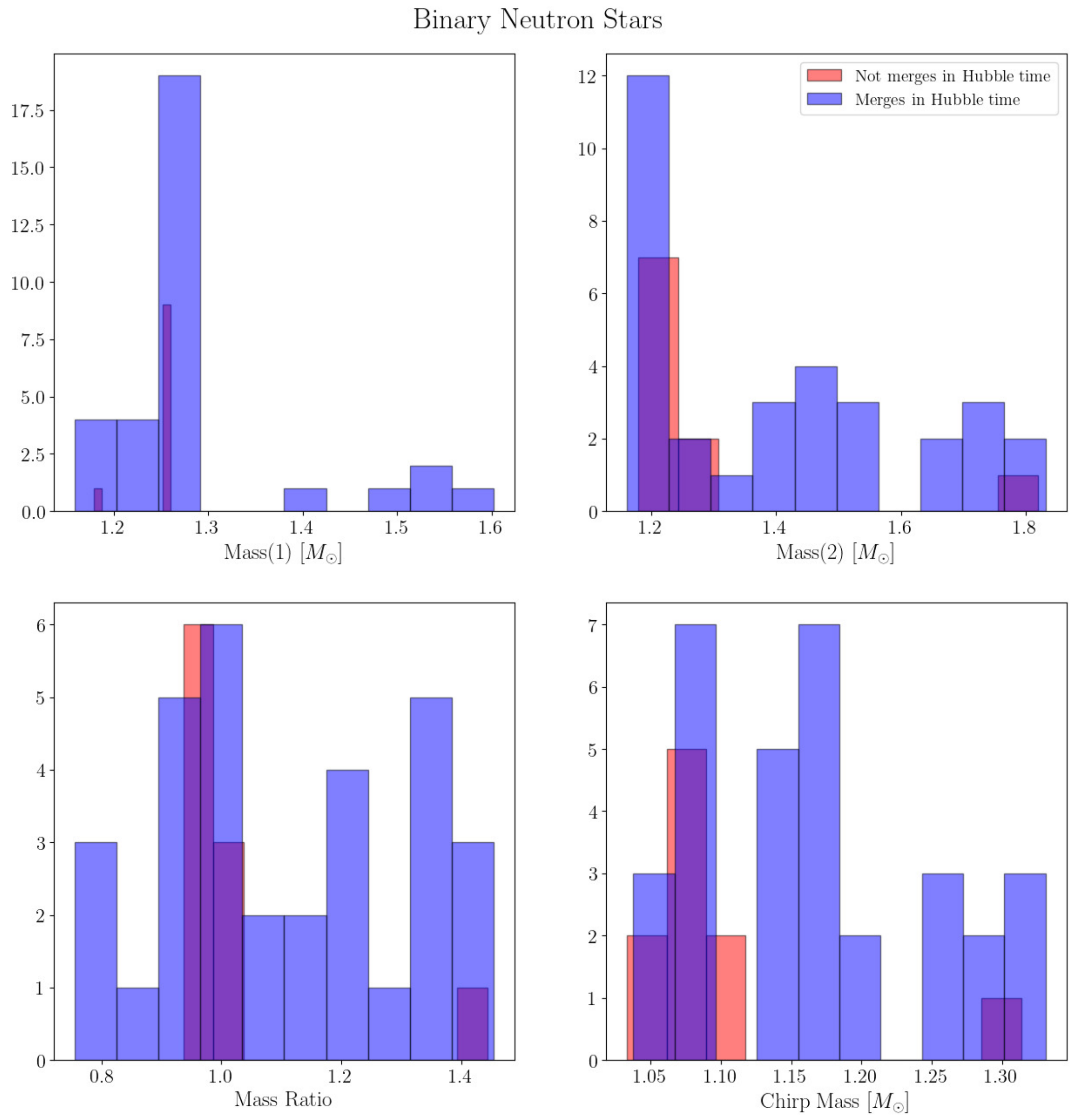
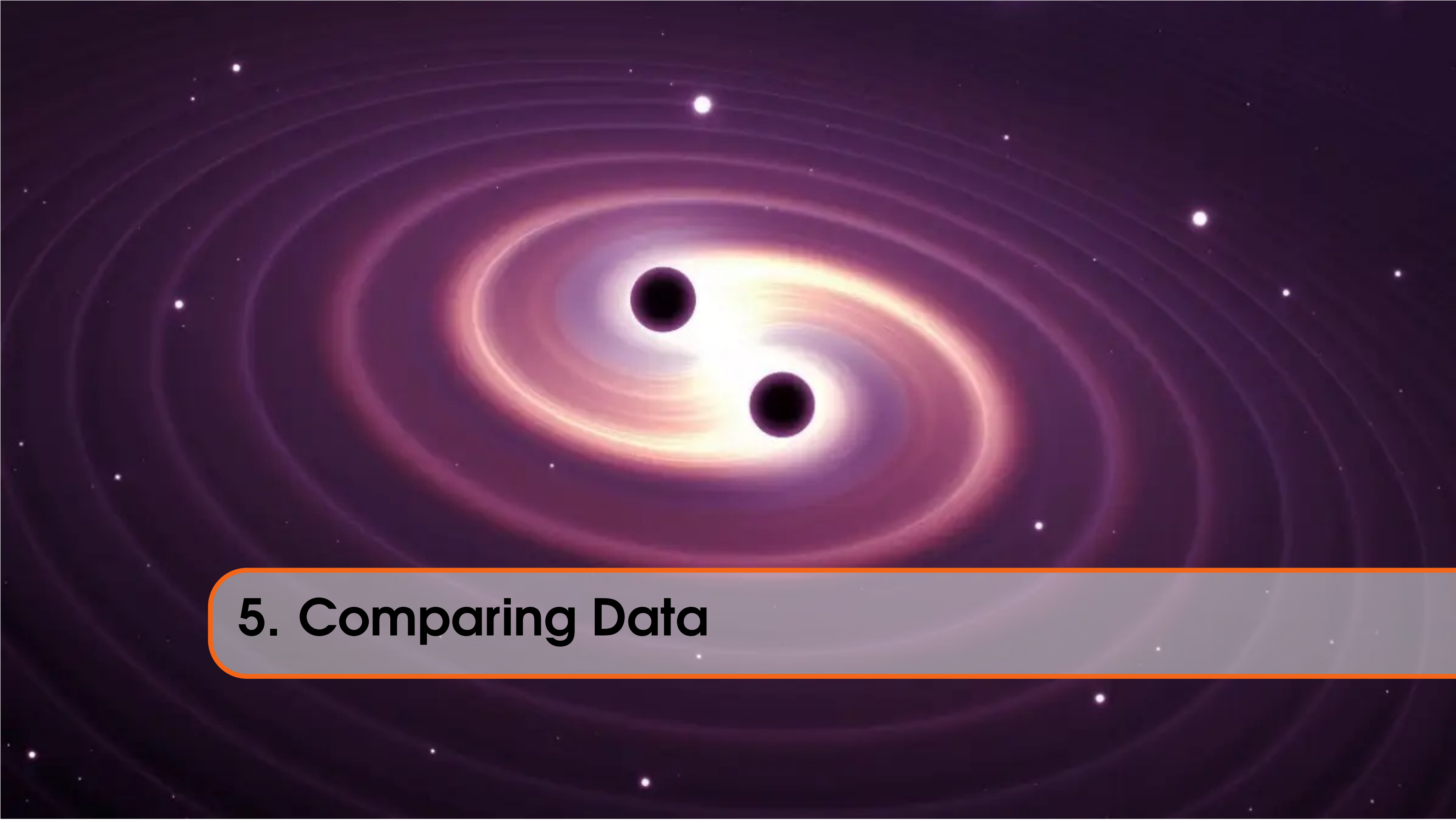


Figure 4.24: Histograms for Binary Neutron Stars

- Binary neutron stars are the rarest type of DCOs. Only 42 among the 4571 DCOs were BNS.
- Majority of these BNS merge in Hubble time.
- The chirp masses vary from 1-1.4 $M_\odot$.

# III

# Simulating LIGO-Virgo Observations with COMPAS



# 5. Comparing Data

For the final task of the project, we have to vary parameters and try to create a chirp mass distribution which is similar to that of the data collected by LIGO-VIRGO collaboration.

## 5.1 LIGO-VIRGO Data

- LIGO and VIRGO are gravitational wave observatories located in US and Italy respectively. The two have collaborated to detect black hole mergers on three occasions.
- The third observing run (called O3) took place from 1 April,2019 to 21 April,2020. It was divided into two parts- O3a from 1 April,2019 to 1 October, 2019 and O3b from 1 November, 2019 to 21 April, 2020. Further observations were halted because of the COVID-19 pandemic.
- The data obtained from this run has been summarised in a paper titled *"Population of Merging Compact Binaries Inferred Using Gravitational Waves through GWTC-3"*.
- According to the paper, the O3 run observed 76 merger events, out of which 72 have been confirmed to be mergers of binary black holes.

## 5.2 Simulations

To compare our distribution with that of LIGO data, we need to have a bigger sample size. So, in this section, the main runs will simulated a total of 1,000,000 binaries. The seed for these million binaries will be the same, so that it becomes easier to observe the changes resulting from manipulating one or more parameters. The seed that was selected for these runs is SEED-9058, that means COMPAS runs binaries from seed no. 9058 to 1009057.

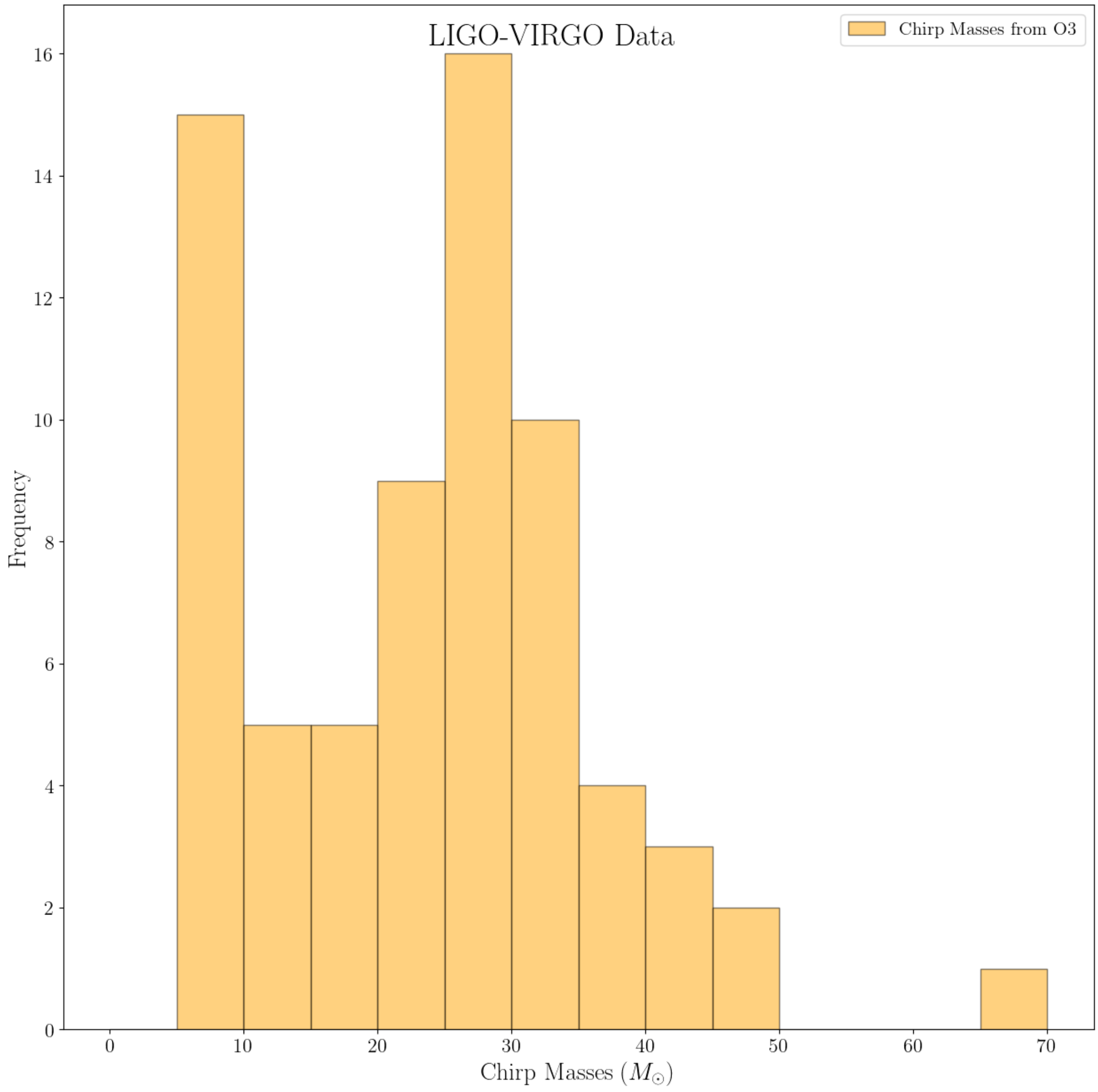


Figure 5.1: Chirp mass distribution of binary black holes from LIGO-VIRGO O3 data

### 5.2.1 Run-1 : Default Parameters

- To get an idea of which parameters to be varied, the first major run was done using default parameters.
- This run took **1 hour** to complete, and generated **3343** binary black holes merging in hubble time.
- The chirp mass distribution of the run is as follows:

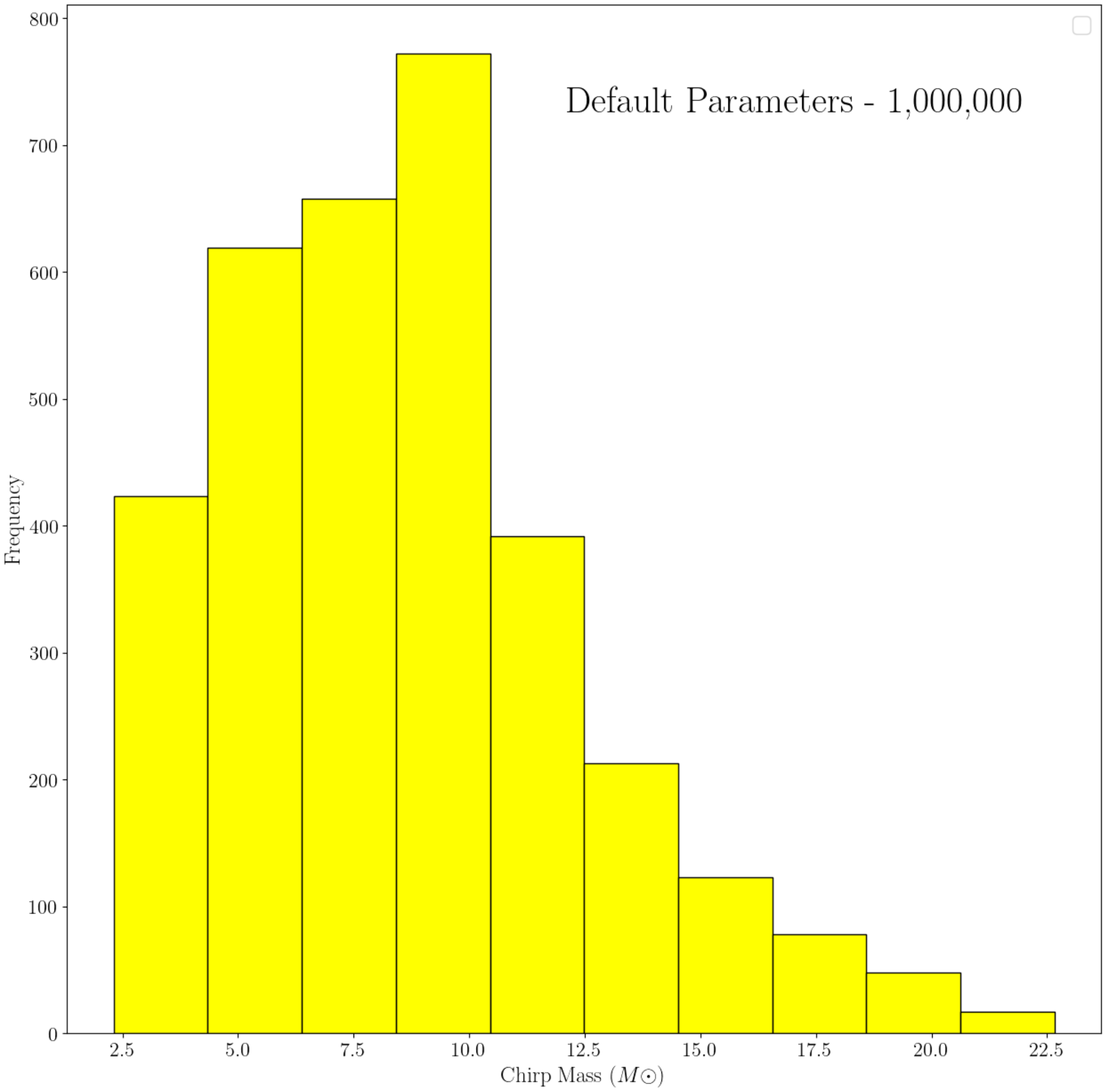


Figure 5.2: Chirp mass distribution of binary black holes obtained using default parameters

**Observations**

- We can see that the peak of the graph is at a much lower value than that of LIGO data.
- Chirp mass seems to increase from 2.5 $M_\odot$, peaking at around 10 $M_\odot$, and then falling off exponentially. The chirp mass is not greater than 22.5 $M_\odot$.
- These values are significantly lower than those of LIGO data.

**Inferences and Adjustment**

- From the first run, we can see that we need to increase the mass of our stars, so that chirp mass distribution is shifted to higher values.
- From amongst the parameters, the natural one to pick at this stage is **–initial-mass-min**. This sets the minimum mass of the primary star. By default, its value is set to 5 $M_\odot$.

### 5.2.2 Run-2 : Changing Mass

- To save time, the number of binaries was set to 20,000. This way, we get a rough idea of how the change in mass affects the distribution and we can quickly adjust it to fit our needs.
- First, the minimum mass was set to 20 $M_{\odot}$. This led to 590 binary black holes being formed with the following distribution.

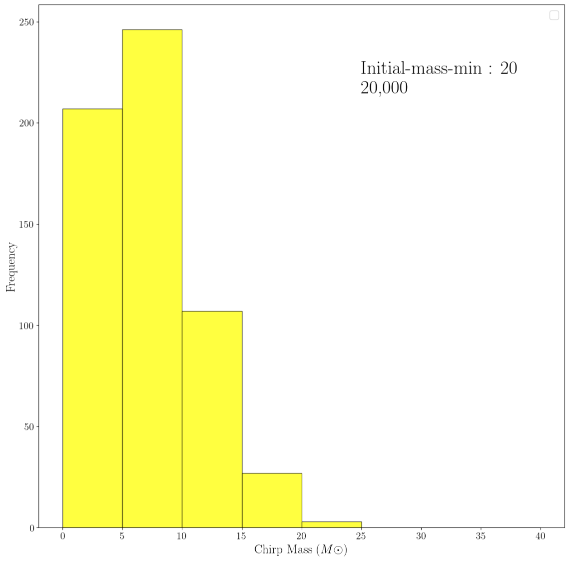


Figure 5.3: Chirp Mass Distribution with initial mass set to 20 $M_{\odot}$ minimum

- We can see that this actually shifted the graph in the wrong direction. So the minimum mass was increased to 50 $M_{\odot}$.

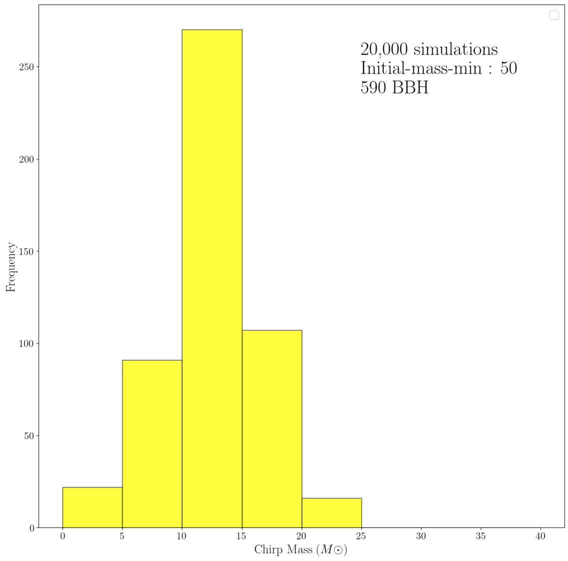


Figure 5.4: Chirp Mass Distribution with initial mass set to 50 $M_{\odot}$

- This distribution is better but still on the lower side.
- From here, the minimum was drastically increased to 100 $M_{\odot}$. However, it still failed to meaningfully shift the curve.

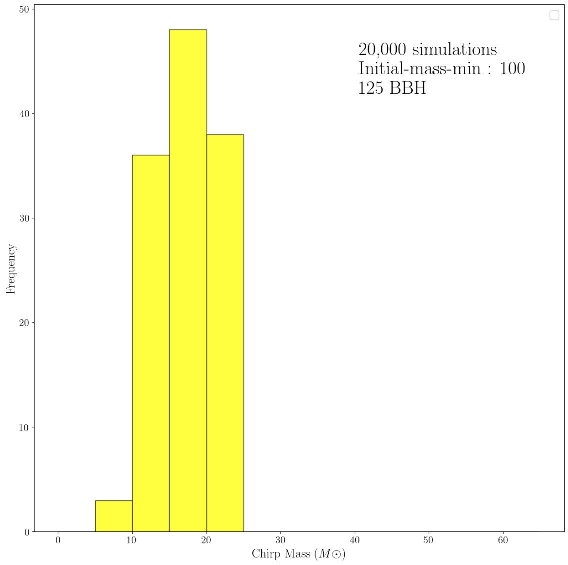


Figure 5.5: Chirp Mass Distribution with initial mass set to 100 $M_\odot$

- Here, we can see that the peak has shifted to 15-20 $M_\odot$. However, the number of BBHs has gone down to only 125. So, more massive systems are not able to merge within hubble time.
- To mitigate this issue, we can take help of another parameter, **–minimum-secondary-mass**. This puts a limit to the minimum mass of the secondary star in the system. The default value of this parameter is 0.01 $M_\odot$.
- Setting minimum secondary mass to 45 $M_\odot$ and minimum primary mass to 75 $M_\odot$ gives the following distribution.

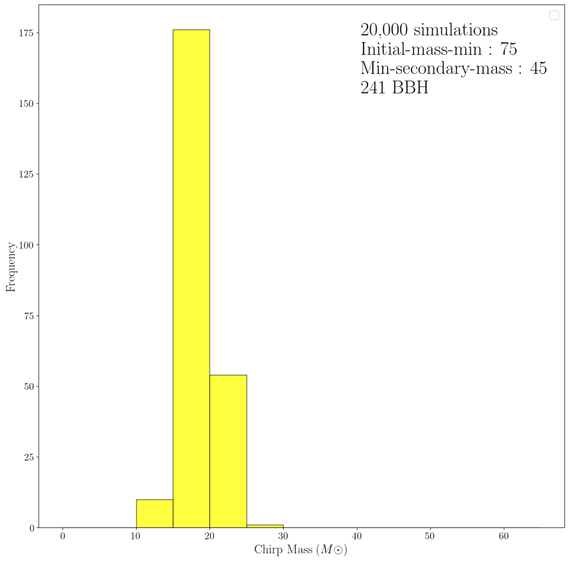


Figure 5.6: Chirp Mass distribution with initial mass min 75 $M_\odot$ and minimum secondary mass 45 $M_\odot$

- We can see that this successfully keeps the chirp masses above 10 $M_\odot$, however the peak is still between 15-20 $M_\odot$.

**Inferences and Adjustment**

- From Run-2, we can conclude that mass by itself won't be enough to give a distribution comparable to LIGO data.
- We need another parameter which can nudge the graph in the right direction.

### 5.2.3 Run-3 : Changing Metallicity

- **Metallicity** measures the abundance to elements in star that are heavier than hydrogen and helium. This include carbon, oxygen and iron.
- By default, the parameter **–metallicity** is set to 0.0142, which is the Solar metallicity, with an upper limit of 0.03 . However, this assumption may not be appropriate if one it trying to re-create LIGO data.
- The mergers detected by LIGO-VIRGO are of black holes which were formed in the early universe, because the time taken from formation of a binary black hole to its eventual merger is really large.
- These black holes would have formed from deaths of early stars. Since these are the stars from very early universe, they will mostly be composed of hydrogen and helium, and be poor in heavy metals. Therefore, the metallicities of these stars would be the lowest of all stars.
- In COMPAS, the minimum value of metallicity that can be set if 0.0001.
- Changing metallicity and keeping other parameters the same gives us the following distribution.

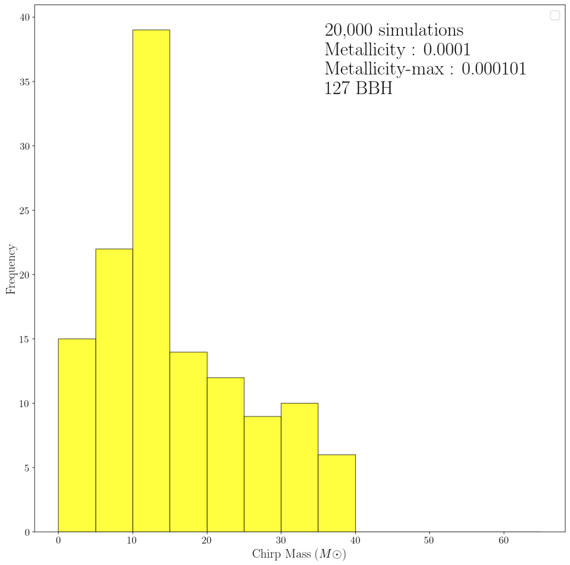


Figure 5.7: Chirp mass distribution using metallicity 0.0001

- We can right away see that even though the peak is around 10 $M_\odot$, this change has allowed the distribution to go upto 40 $M_\odot$.

**Inferences and Adjustment**

- Changing metallicity promises good results towards trying to replicate the LIGO data.
- If we use low metallicity and higher masses simultaneously, the distribution might look similar to observed distribution.

### 5.2.4 Run-4 : Changing Mass as well as Metallicity

- Now we have enough confidence in the parameters we want to change, so we can run a 1,000,000 binary run.
- For this run, the parameters were set as follows:
    - Initial mass minimum : 50 $M_\odot$
    - Minimum secondary mass : 30 $M_\odot$
    - Metallicity : 0.0001
    - Maximum metallicity : 0.000101

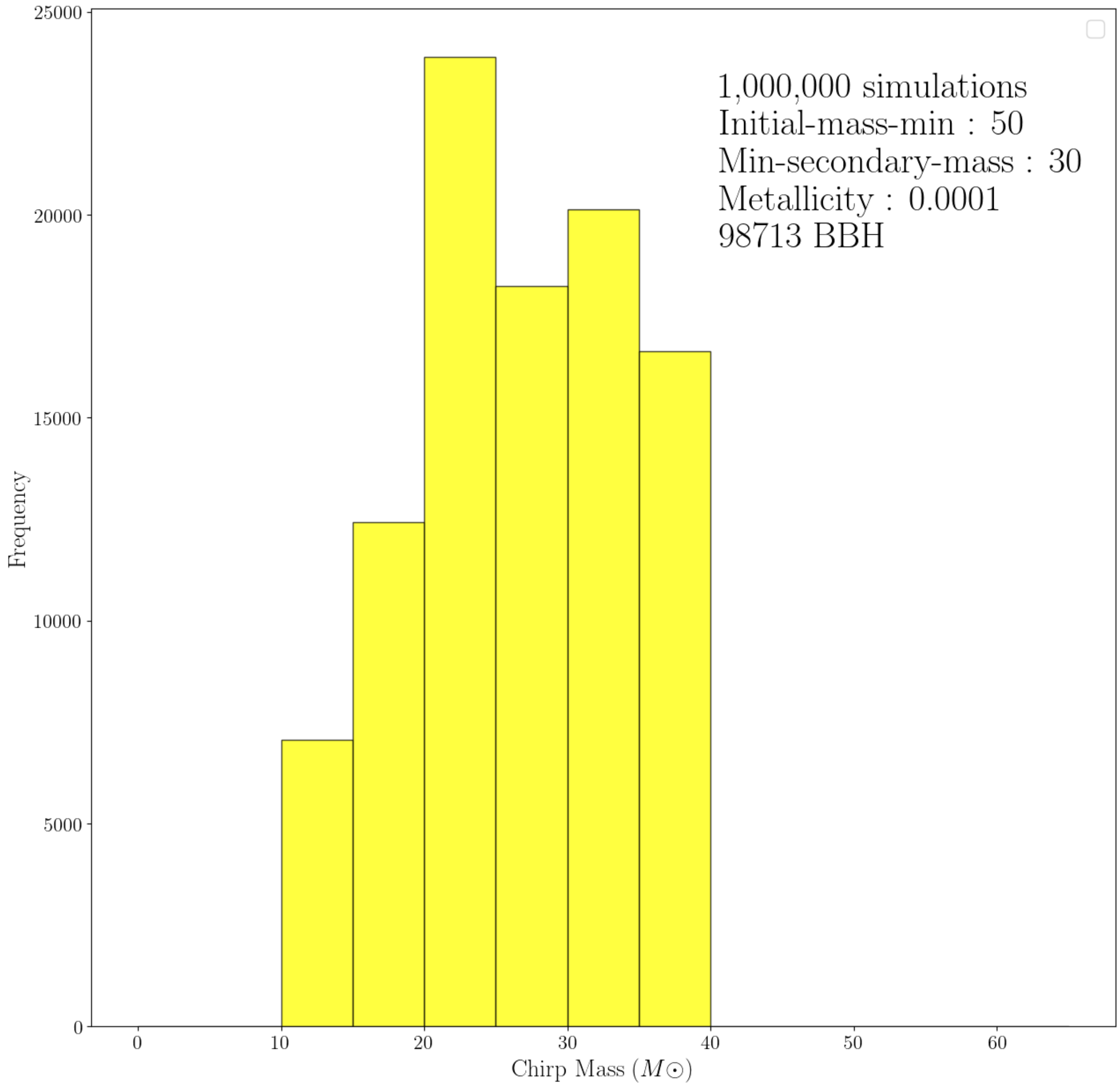


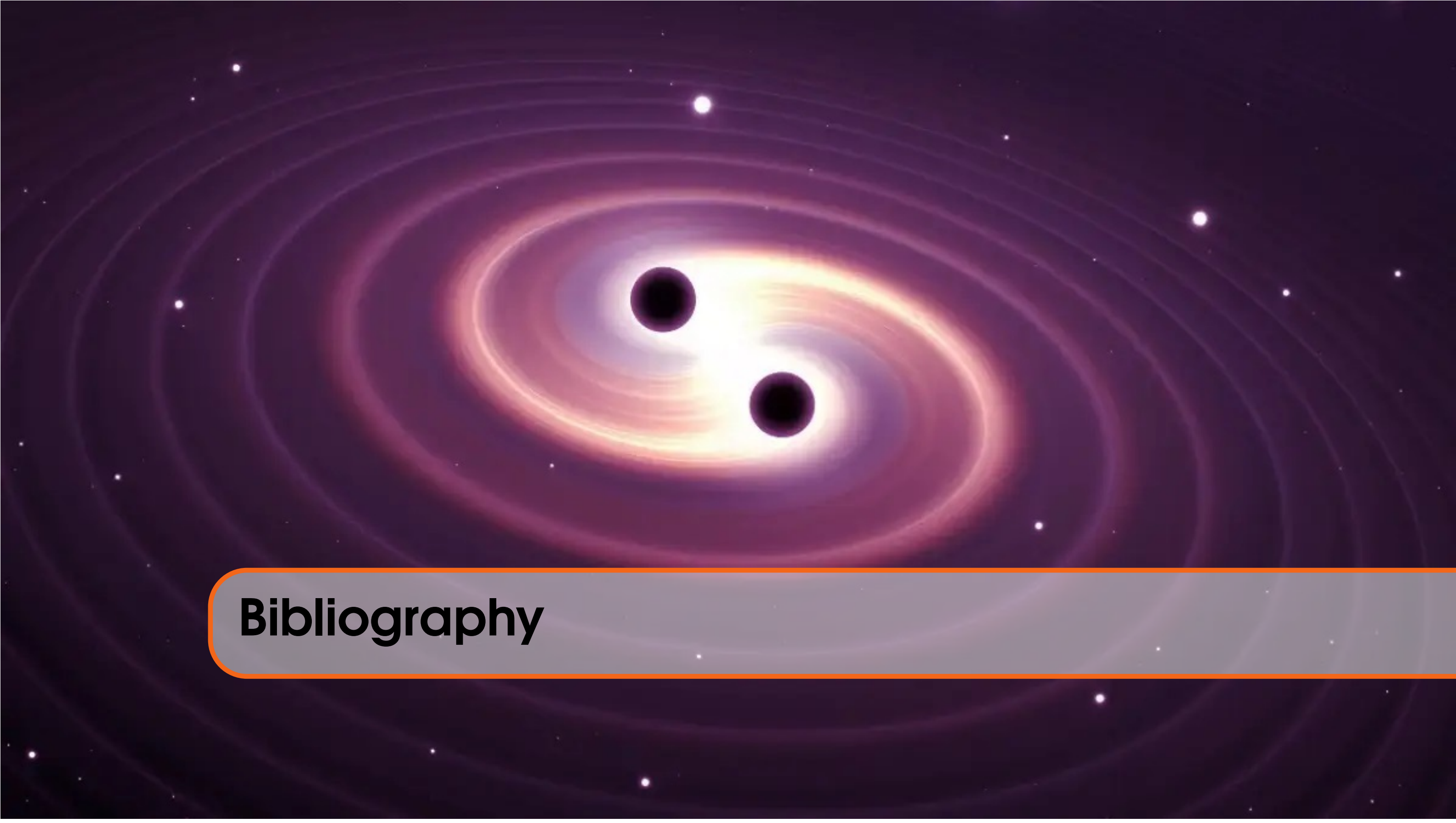
Figure 5.8: Chirp Mass distribution by changing initial masses and metallicity simultaneuosly

**Observations**

- We can see that this distribution is a pretty good fit to the LIGO data, by changing only two parameters.
- The chirp mass increases from 10 $M_\odot$, peaking in the range of 20-25 $M_\odot$ and reducing in number towards 40 $M_\odot$.

## 5.3 Discrepencies between LIGO data and simulations

- The final generated plot, though a good replication, is not a perfect fit to the data observed by LIGO-VIRGO.
- We can observe that in none of the simulated plots does the chirp mass distribution cross the mark of 40 $M_{\odot}$.
- This could be due to the fact of the hard upper limit on initial mass set to 150 $M_{\odot}$, due to which systems with higher chirp mass were not able to evolve.
- Another explanation might be that not enough parameters have explored in this report which can provide a better fit to the data.
- Nonetheless, these simulations do give us an insight into how massive the early stars must have had to be in order to produce binary black holes this massive enough, and still able to merge within the age of the universe.



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- Abbot R. (2023), Population of Merging Compact Binaries Inferred Using Gravitational Waves through GWTC-3, 2–3

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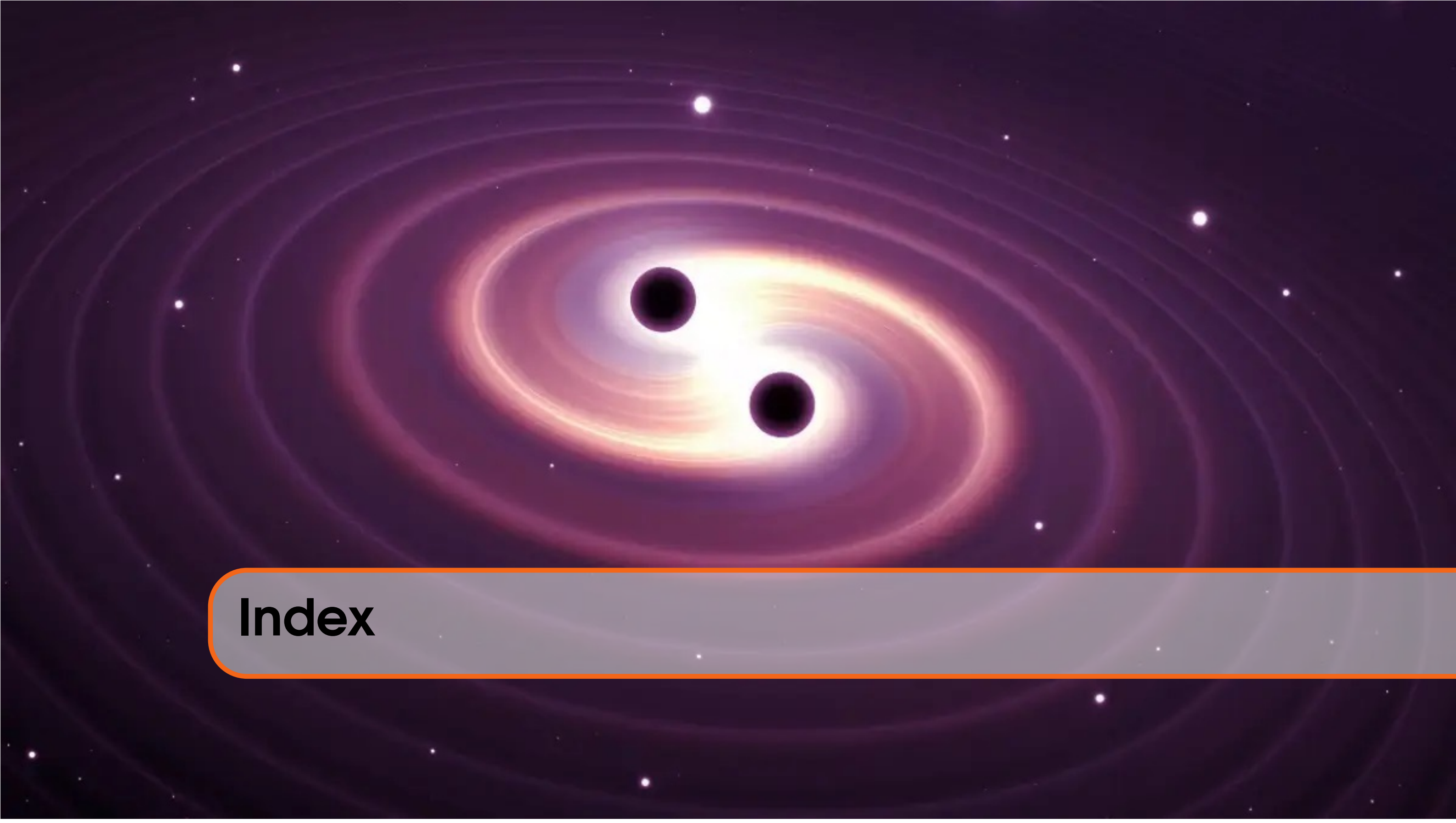

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