Krittika Summer Projects 2024

Binary black holes from scratch

Sriram R

KRITTIKA SUMMER PROJECTS 2024

Binary black holes from scratch

Sriram R[1]

[1]Indian Institute of Science, Bangalore, India

PUBLISHED BY KRITTIKA: THE ASTRONOMY CLUB OF IIT BOMBAY
GITHUB.COM/KRITTIKAIITB
First Release, August 2024

Abstract

Within the duration of the Krittika Summer Project 5.0 program in the Summer of 2024, concepts related to Stellar Evolution and Compact Object Mergers were studied. The rapid simulation tool "Compact Object Mergers: Population Astrophysics and Statistics" (COMPAS) was utilized for further analysis and understanding of learned concepts.

Contents

Theory

1. Stellar Evolution

1.1 Overview

Stars are born from gravitational collapse of dust and gas. They fuse hydrogen into helium in their core for a majority of their lives (called the Main Sequence). Their evolutionary paths depend on their starting mass, but all stars eventually "die" or stop being "stars". The remaining matter from some forms of star deaths can condense into or merge with other gas and dust clouds and eventually give rise to new stars. The timescales involved in this overall process is very large so we cannot directly observe this happening. We rely on other methods such as predicting the internal structure and processes happening within the stars to model and understand their evolution, and then try to fit our observation of various stars in different stages of their life cycles and see if our understanding was correct.

Variable stars are stars whose properties such as brightness, radius, spectra etc change over time. Due to stellar evolution, all stars are technically varying over time but we use the term "variable stars" primarily for stars that show such variations within observable time scales.

Plotting star brightness against colour, the Danish astronomer Ejnar Hertzsprung observed that most stars fell into certain regions on the plot. This diagram came to be known as the Hertzsprung-Russel diagram, or the H-R diagram. It was discovered that these regions show a star's progression through its evolution. Stars of given masses and ages were present in predictable regions on the HR diagram. Hence the general idea came about to be that on identifying a variable star, it is studied and its properties are measured. Using these measurements and observations of the way the variable star's properties change over time and by identifying the region on the HR diagram that the star belongs to, we can generalize the properties to other stars in the same region as well.

1.2 Regions of the H-R diagram

The regions of the H-R diagram correspond to periods of a star's life where it has properties that are defined by the region it belongs to. Not all stars pass through all regions through their lifetime. The path they take through the H-R diagram depends on the mass that they start their life with.

1.2.1 Protostars

Newly formed stars are called protostars or pre main sequence stars. Orion and Taurus constellations are home to "star-forming regions", which are concentrated gas and dust clouds. One of these regions is the Orion nebula (Messier 42). We have observed many young variable stars in these clouds, which were initially called Orion variables or nebular variables. T Tauri Stars (TTS) are PMS stars that are variable and have accreting gas and dust around them. The star's variability is primarily due to accretion. The accreting gas and dust also give off heat and light due to friction as it accelerates toward the star. This accretion disc is also called "circumstellar disc".

FU Orioinis (FUor) or UX Orioinis (Uxor) are two other classes of PMS stars. FUor stars get mass abruptly transferred from their accretions discs so they display extreme increases in brightness (for example, V1057 Cyg went around 6 magnitudes brighter in one year) and then spend years dimming. UXor stars show irregular fluctuations in brightness, probably because of clumps in their accretion discs obstructing the star from time to time.

1.2.2 Main Sequence Stars

PMS stars eventually gain enough mass to intitiate fusion of hydrogen in their cores. This marks their transition to zero-age main sequence stars. More massive the star, faster it gets over the main sequence; the larger mass leads to larger pressure at the core, thus faster nuclear reactions. Stars in the main sequence might have very high pressures and temperatures in the cores relative to human standards, but the atoms inside are still distinct atoms, and matter still follows an equation of state such as the ideal gas law.

Changes during the main sequence (compositional changes leading to structural changes) take place on large timescales so we cannot observe them directly. But even observing current structural states can be hard since they happen inside. We indirectly do this through astroseismology – studying sound and gravity waves on the surface to infer what's going on underneath. These waves/vibrations on the surface can be detected as localised luminosity changes – called pulsations. Delta Scuti Stars have 1.5 to 3 times the mass of the Sun, and have dozens of pulsation modes with brightness changes of large amplitudes associated with them. By studying pulsations in many stars of different ages, we can finetune our understanding of the internal structure of stars and how they vary over time.

Other types of variability are spots and flares, both caused by magnetic field variations. Spots are caused by magnetic field obstructing convection of energy from inside to outside, causing that region to dim, while the opposite happens in flares. Some spots may survive for several rotations and thus can be observed as periodic dimming of stars. UV Ceti stars have intense magnetic fields and cooler, dimmer surfaces so when there are flares, they are easily detected. Our sun has a 22-year cycle where every 11 years the sunspot activity waxes and wanes and reverses magnetic polarity. We don't know why.

1.2.3 Post Main Sequence Stars

When the hydrogen in the core is depleted, the nuclear reactions in the core stop temporarily, and complex processes lead to the star becoming a red giant: diameter increases, luminosity increases, temperature decreases. Stars a few times more massive than the sun will cross the instability strip as they leave the main sequence and enter the post main sequence stage. These are called Cepheid Variables, and they follow the Leavitt Law (Period-Luminosity or PL relation): the period of pulsations is proportional to the mean absolute luminosity. This can be used to determine the distance between Earth and a given Cepheid Variable, and thus also the distance to the galaxies that they belong to. Other variable classes such as the Scuti and RR Lyrae pulsate for similar reasons but have their own PL relations. Stars outside the instability strip can pulsate as well, but they mostly don't have well defined periods or large amplitudes.

In the last stages when the star is about to run out of nuclear fuel, most stars become what are called Asymptotic Giants (AGB stars – B for branch). Evolved red giants that have begun helium fusion in the core are in the horizontal branch, but once the core helium is over, they move onto the Asymptotic Giant Branch. The nuclear reactions in AGB stars do not happen in the core, but rather in layers closer to the surface. Their cores are primarily made of Carbon or Oxygen, but this core is surrounded by Helium and Hydrogen layers, where fusion can still take place. Since the fusion takes place closer to the surface, the luminosity of these stars is much higher, usually among the brightest stars. But the star also increases in radius a lot, so they are much cooler on the surface – they are red. In later stages, AGB stars also have "thermal pulses" when a layer of helium suddenly undergoes fusion, leading to drastic variations in the star. These are predicted but not proven yet, though it is possible we have already witnessed them in our years of observations.

Some stars in the AGB are called Mira variables. They have large radii and large amplitude pulsations (up to 10000 factors in brightness), and may take hundreds of days to complete pulsations. Mira variables have a P-L relation of their own so they can be used to indicate distances to galaxies. They also have high mass loss rates – they are the source of most if not all processed interstellar materials, including us. The period of Mira variables depends on their sizes, so contractions and expansions can be studied.

Beyond the AGB, the last stage may be RV Tauri stars. They are so bright for their sizes that they cannot maintain regular period of pulsations (why?). They have chaotic periods. RVa stars and RVb stars – former stars have almost constant mean magnitudes but latter have a secondary period through which they slowly get dimmer. RV Tauri stars are headed towards becoming planetary nebulae or white dwarfs. The reason why some are RVa and some are RVb is not proven yet.

As the star evolves, the core material starts to degenerate- the atoms are so compressed that their electric fields are not able to keep them separate. Now they follow a degenerate equation of state. This degenerate core blows away the outer layers, leaving the core alone remaining, called a white dwarf. The blown away material remains lit by remnant material and becomes diffused around the star, called a planetary nebula.

1.2.4 Stellar Death

The Chandrasekhar limit is an important critical value – it is the mass above which a star becomes a neutron star and goes out as a supernova and below which it becomes a white dwarf. If the mass is above this limit, the star's gravity wins out over the electron degeneracy pressure, thus causing an implosion. Stars with lower mass (less than 8 times the mass of the sun) lose their mass more easily through stellar winds and such, thus preventing the implosion. Over 99 percent of stars in the universe meets the criteria to become a white dwarf, as opposed to a neutron star/supernova.

Post AGB (pAGB), there can be slow evolutionary changes caused by the fact that nuclear fuel burns faster as pressure and temperature increases. There are fast changes such as thermal pulses when suddenly a layer of helium starts burning. These two together can cause blowing off of material from outer layers leading to accumulation of dust around the star, leading to obscuration. Examples of extreme late stages include R CrB stars which have periodic obscurities caused by the dust clouds either being generated freshly or due to an already generated dust cloud orbiting the star. Another example is FG Sagittae, whose weird variations probably still remain a mystery (check it out).

Once all the enveloping material has been blown away, we are left with the white-hot core made of Carbon and Oxygen, which we call a white dwarf. These are only a few kilometres wide and do not glow by any reactions; they simply are cooling down from the heat leftover from their lives as stars. White dwarfs also undergo pulsations, but their period can be as small as minutes. (Now they are talking about astroseismology pulsations?) Studying the pulsations in white dwarfs has helped us learn about high density matter. The coldest of the white dwarfs help us put an upper limit to the age of the universe itself.

If the star's mass was above the Chandrasekhar limit, once it exhausts its hydrogen and helium, it proceeds to fuse carbon into heavier elements and so on, until it reaches iron. Since up intil iron, fusion is exothermic, you only need enough energy to start the reaction, and it goes on. But fusion of iron is endothermic, so it stars drawing energy from the surrounding core material to begin. This energy is what has been keeping up the pressure required to hold out against the gravity pulling the outer layers in, so now as it depletes, gravity wins. The violent implosion leads to a supernova, releasing the pent up gravitational potential energy, creating every element in the periodic table, and a storm of subatomic particles traveling at close to the speed of light.

What's leftover from such an event depends on the mass. If the mass is less than 3 solar masses, the remaining dense mass is a neutron star only around 10 kilometres across (extreme density). They can spin several times a second and release bursts of radiation. Read pulsars and magnetars. If the mass is over the limit of 3 solar masses, it forms a black hole, the most extreme object in the universe, whose escape velocity is higher than the speed of light.

2. Binary Systems

2.1 Overview

Binary systems (and in general multiple star systems) are interesting primarily due to the fact that some processes like eclipsing, doppler shifting and orbital period changes help us study the members involved to a higher degree than if they were standalone. But something else than can happen is mass transfer. The Roche limit is the equipotential line where an object is equally pulled by the gravity of two stars in a binary system. Crossing the Roche limit (usually due to one star growing past the boundary as it evolves) leads to mass transfer. The recipient gets its mass increased midway through its evolution, leading to some interesting evolutionary pathways.

2.2 Binaries

Now consider in a binary system, one member is a regular star and the other is a compact object: a white dwarf, a neutron star, or a black hole. The donor is the former and the recipient is the latter, which are collectively called "cataclysmic variables" as the mass transfer causes tremendous increases in luminosity.

2.2.1 White Dwarf Binaries

White dwarf binaries are the most common cataclysmic variables. Dwarf novae are composed of one white dwarf and one main sequence star. Mass gets pulled from the latter towards the former. There are irregular variables in the white dwarf's luminosity as the mass impacts its surface, but sometimes the accretion is fast enough that the accreting mass itself starts glowing (as it does in protostars) called an "outburst". The frequency of these outbursts increases with increasing mass transfer rates. When the accretion disc gets stuck in the outburst mode, leading to a period of high brightness, it is called a "standstill". Sometimes, the mass transfer rates are so high that the accretion disc remains permanently bright. Example is TT Arietes, which is permanently in outburst except some very rare unexplained dips in magnitude.

As mass accumulates on the white dwarf (typically hydrogen and helium from the other star's outer layers), it may gain enough mass to start fusing the acquired mass, thus making it a new star of its own, called a classical nova (nova means new in latin), brightening by a factor of 10000. Most novae are recurring but over very large periods. But some recur with periods of years or decades and are called recurring novae. Each time it goes nova, the dwarf's mass gets pushed closer and closer to the Chandrasekhar limit, until it reaches it and goes supernova, brightening by a factor of billions of times. This has not been observed yet (why).

Sometimes, the magnetic field of the white dwarfs interfere with accretion so no accretion disc is formed; instead, accretion happens along lines of mass leading into the poles of the dwarf. These are called polars. Other times, the mass accretion doesn't happen regularly, due to very wide orbits. In these cases, accretion happens due to stellar winds pushing matter from the star onto the dwarf irregularly, causing slow changes in brightness with sudden intermittent increases (probably from the dwarf going nova or due to change in accretion rates).

2.2.2 X-Ray Binaries

Neutron stars and black holes arise from very massive stars dying, so its very rare to see binaries where the primary is one of them. Binaries where the primary is a neutron star or a black hole instead of a white dwarf are prominent in X-ray instead of optical light, and are called X-ray binaries. Accretion material from the star travels so fast as it gets closer to the primary (significant fraction of speed of light) that it emits X-rays on impact.

II COMPAS

3. Stellar Evolution using COMPAS

3.1 COMPAS

Compact Object Mergers: Population Astrophysics & Statistics (COMPAS) is a population synthesis tool for simulating and studying the evolution of single stars and binaries. It enables the controlled and rapid simulation of massive populations to gain statistical insight into stellar evolution.

Reading the documentation of COMPAS, it is simple enough to get started with simulating stellar evolution and analysing the obtained results. Beyond the work that has been presented here, COMPAS as a tool seems valuable to the learning of astrophysical processes and phenomena since it can be used to synthesize populations to verify and further our understanding of the same.

3.2 Initial Test Run

star_1_init_mass	star_2_init_mass	star_1_final_mass	star_2_final_mass	system_init_mass	system_final_mass	system_loss
5.192743353	4.691859827	1.016539444	0.707439246	9.884603179	1.723978691	8.160624488
19.2536479	6.727545657	5.192339714	6.725146813	25.98119356	11.91748653	14.06370702
9.216274884	8.05089177	1.216020661	1.26	17.26716665	2.476020661	14.79114599
5.182262098	4.988817029	1.281014801	0.98620501	10.17107913	2.267219812	7.903859315
22.2782465	19.533601	4.768245022	11.70912159	41.8118475	16.47736661	25.33448088
14.22064847	1.912375489	3.455946916	1.912375489	16.13302396	5.368322405	10.76470155
14.15423473	3.582383322	1.368748172	0.87155865	17.73661805	2.240306822	15.49631122
5.761156159	2.989310618	0.792900318	1.326914158	8.750466777	2.119814476	6.630652300
7.218980344	2.693180014	7.18009392	2.693180014	9.912160358	9.873273934	0.038886424
6.234425246	3.750298686	1.203371087	0.842306154	9.984723932	2.045677241	7.939046690

Figure 3.1: Results of evolution of 10 random Binary Systems

Figure 3.1 displays the data obtained from a randomly initiated COMPAS simulation of 10 binary systems. This was done to understand the basic usage methodology of COMPAS and initialization parameters.

The systems have 100 percent accretion, meaning all mass lost from one star to the other results in accretion. This indicates that the total mass of the system must be conserved aside from the small drop caused by nuclear fusion.

However, a large discrepancy in the total mass is observed. This is understood to be caused by other sources of mass loss such as stellar winds and supernova explosions.

3.3 Detailed Evolution

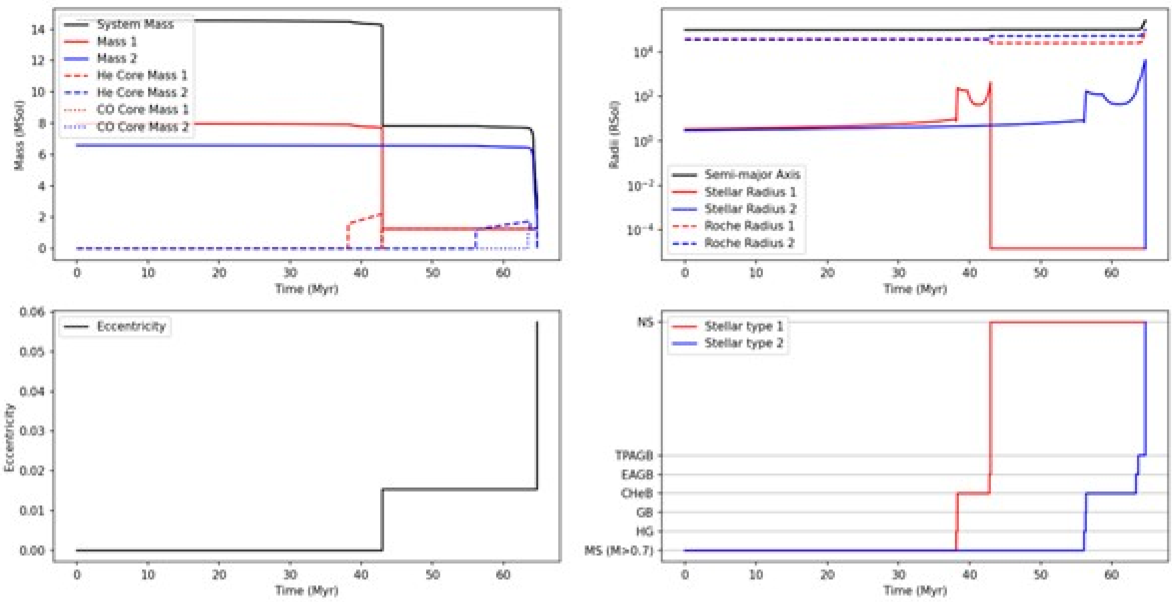

Figure 3.2: Results of detailed evolution of one Binary System

Figure 3.2 displays the visualized variation of parameters over the course of the lifetime of one binary system simulated using COMPAS. A detailed timeline of events observed is presented below:

38.15 Myr:
Star 1 gradually evolves from Main Sequence (M>0.7) to Hertzsprung Gap. This coincides with a sudden increase in the mass of the Helium core of star 1, as well as a slight dip in the stellar radius of star 1 before it begins to rise sharply.

38.27 Myr:
Star 1 evolves from Hertzsprung Gap to "First" Giant Branch. The radius continues to increase sharply and while the increase in the mass of the Helium core slows down to a more gradual pace, the total mass of the star shows a slight but non-insignificant reduction.

38.30 Myr:

Star 1 evolves from "First" Giant Branch to Core Helium Burning. The total mass of the star begins a slow but steady decline, while the radius which was increasing rapidly suddenly halts and shows a slight decrease. Helium core mass continues to gradually grow in mass.

39.97 Myr:
Star 1 shows a smooth decrease in radius before increasing steadily. This change is apparently not accompanied by any other changes?

42.80 Myr:
Star 1 evolves from Core Helium Burning to Early Asymptotic Giant Branch, accompanied by a sharp increase in its CO core mass.

42.97 Myr:
Star 1 evolves from Early Asymptotic Giant Branch to Neutron Star, drastically decreasing in radius (from around 400 times larger to 5 magnitudes smaller than the Sun's radius). It loses mass rapidly and goes down from 7.67 times the mass of the sun to 1.27. The Roche Lobe radii of both stars were similar until this point (38000 and 35000 times the solar radius for star 1 and star 2) but now, the Roche Lobe radius of star 2 shoots up to 51000 and Roche Lobe radius of star 1 goes down to 24000. The system's eccentricity increases to 0.015

56.10 Myr -> Star 2 gradually evolves from Main Sequence (M>0.7) to Hertzsprung Gap.
56.31 Myr -> Star 2 evolves from Hertzsprung Gap to "First" Giant Branch.
56.38 Myr -> Star 2 evolves from "First" Giant Branch to Core Helium Burning.
63.40 Myr -> Star 2 evolves from Core Helium Burning to Early Asymptotic Giant Branch.
63.73 Myr -> Star 2 evolves from Early Asymptotic Giant Branch Thermally Pulsing AGB, showing a sudden drop in the Helium core mass while still in this phase. This was not observed in star 1.
64.71 Myr -> Star 2 evolves from Thermally Pulsing AGB to Neutron Star.

Notes:

1) The primary difference between the two is their initial masses, which are still close enough to follow an almost similar evolutionary path. It is noted that as expected, the star with slightly higher mass (8 Solar masses) progresses through its evolution much quicker compared to the other star with slightly lesser mass (6.6 Solar masses), so much so that the heavier star finishes its entire evolutionary path before the other star leaves its Main Sequence.

2) While the star is in the Hertzsprung Gap, its Helium core increases in mass, until it reaches the Giant Branch where it soon begins fusion of Helium in its core. In this process, the Helium core mass keeps increasing due to Hydrogen burning in outer layers which leads to Helium deposition on the core.

3) Despite the mostly similar evolutionary paths, star 2 enters the Thermally Pulsing AGB phase whereas star 1 directly went from Early AGB to Neutron Star.

4) Just before becoming a Neutron star, the swelling in size of star 1 is up to 440 times the size of the sun, whereas for star 2, its about 4400 times.

5) The supernova explosion of star 1 causes the orbit's energy to change, thus leading to a drastic increase in the eccentricity at that point.

3.4 10000 Binaries

A simulation was run with default settings for the fraction of accretion (1.0). This means that mass transfer is fully conservative, no mass that gets ejected from one star is lost without accretion on the other star. Some observations are highlighted below:

CPU time for simulation: 302 s
Fraction of systems with no mass transfer: 0.4148
Fraction of systems with stable mass transfer: 0.1327
Fraction of systems with unstable mass transfer: 0.4525
Fraction of systems with stellar mergers: 0.3109

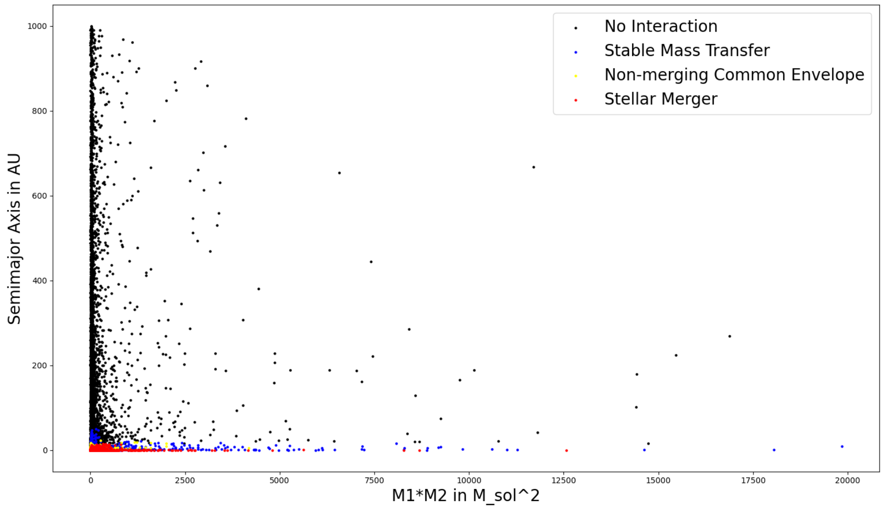

Figure 3.3: Mass product plotted against Semi Major Axis

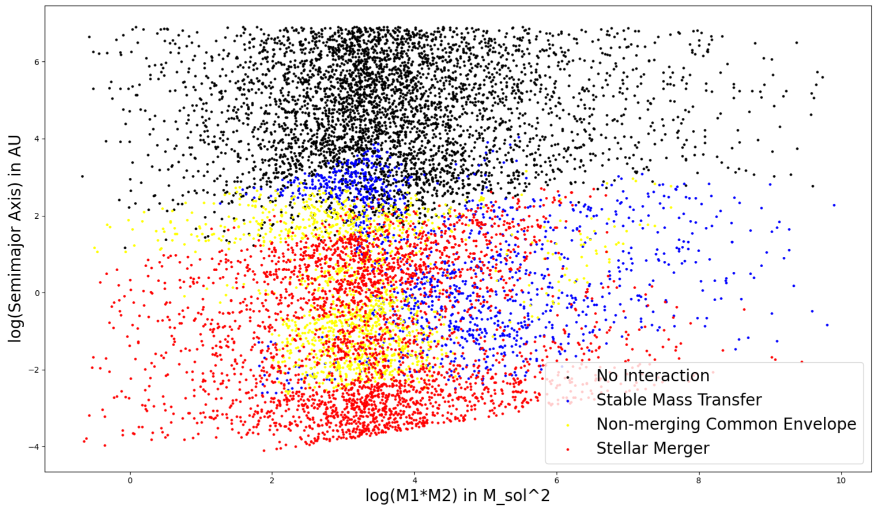

Figure 3.4: Log(Mass Product) plotted against Log(Semi major Axis)

A series of plots that were derived from the output data of the 10000 binary simulation are presented here. Figures 3.3 and 3.4 show that as compared to directly plotting quantities, using a logarithmic scale gives a clearer picture of the pattern or distribution.As expected, it is clear from Figure 3.4 that as the initial separation between the two stars (inferred from the semi major axis of their orbit) increases, there is less possibility of any sort of mass transfer interaction between them.

In a similar vein, it is noted that just as expected, as the initial separation decreases, there is an increasing chance of there being a stellar merger, and at very low values, a stellar merger is inevitable. Curiously, it is observed that for mid range values of semi major axis, for higher mass products (more massive stars involved), there seems to be a higher preference for stable mass transfer, over common envelopes or stellar mergers.

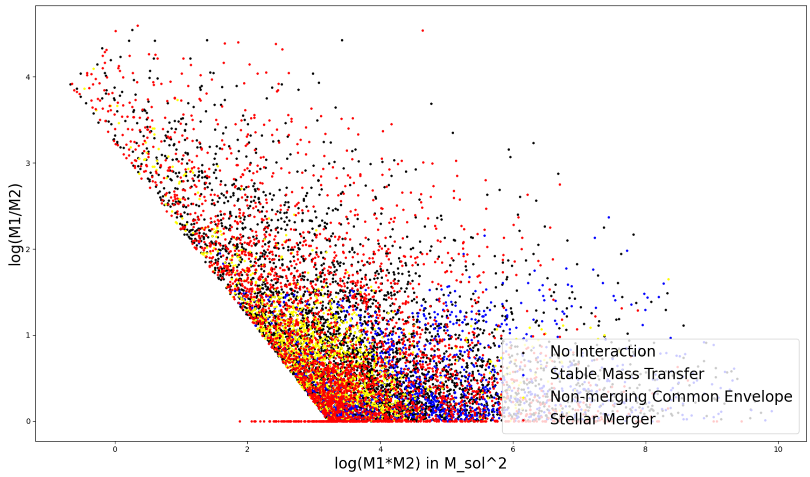

Figure 3.5: Log(Mass Product) plotted against Log(Mass Ratio)

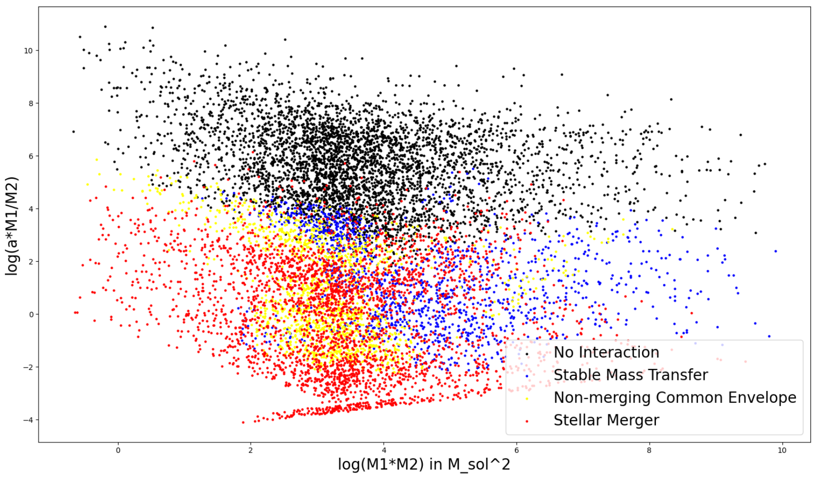

Figure 3.6: Log(Mass Product) plotted against Semi Major Axis * Log(Mass Ratio)

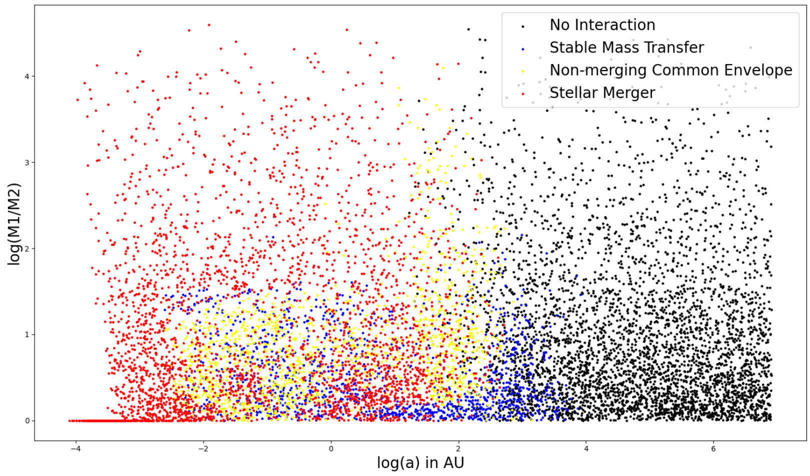

Figure 3.7: Log(Semi Major Axis) plotted against Log(Mass Ratio)

Figures 3.5 to 3.7 outline efforts undertaken to visually separate the regions of different types of stellar interactions based on different combinations of initial parameters. It is noted that plotting log of semi major axis against the log of mass ratio seems to have the best separation of these attempts.

3.5 10000 Binaries: 0.5 Fraction of Accretion

A simulation was run with custom settings for the fraction of accretion (0.5). This means that mass transfer is not fully conservative, 50 percent of the mass that gets ejected from one star is lost without accretion on the other star. Some observations are highlighted below:

CPU time for simulation: 297 s
Fraction of systems with no mass transfer: 0.4161
Fraction of systems with stable mass transfer: 0.1329
Fraction of systems with unstable mass transfer: 0.451
Fraction of systems with stellar mergers: 0.3094

It was noted that the spread or distribution of different types of mass transfer interaction was not heavily affected by the change in fraction of accretion. It is possible that certain other outcomes (such as distribution of possibility of supernovae) could be affected by said change.

Nevertheless, from the highlighted observations above, it is clear that due to the lower fraction of accretion, the fraction of systems with no mass transfer has slightly increased.

4. Merge Rates

From the obtained simulation results, there is a need to extract useful information such as Merge Rates as a function of Masses involved. To do so, the Merge Rates are calculated against Chirp Masses, which are defined as:

$$M = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}}$$

Since the distribution is non-uniform, calculating rates is not direct. Intervals need to be defined, similar to the process of generating an histogram. The right number of bins depends on the distribution of data within the results and needs to be found manually for each result.

For each such bin, the number of mergers is divided by the total number of systems within the bin and thus the discretized rate is obtained. As the number of data points within each bin is increased, the rate function will get smoother. For the purpose of this report, 10000 systems are simulated and the resulting systems with double compact objects are studied as described above, to gain insights into relationship between chirp masses and rate of double compact object mergers.

4.1 NS-NS Merge Rates

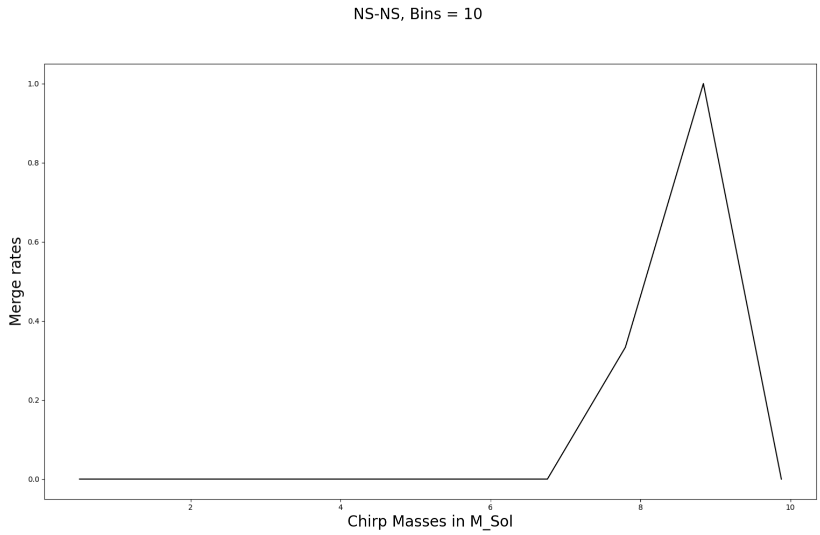

Figure 4.1: Merge rates vs Chirp masses for NS-NS systems

4.2 BH-BH Merge Rates

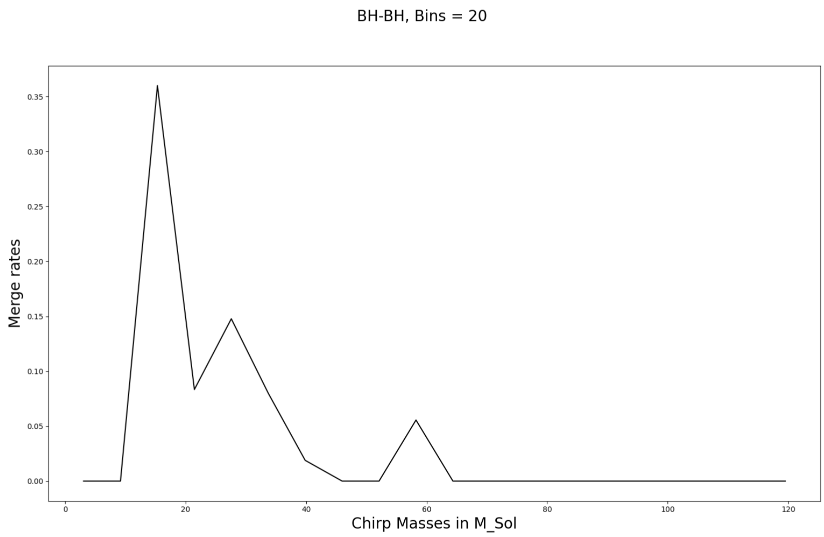

Figure 4.2: Merge rates vs Chirp masses for BH-BH systems

4.3 NS-BH Merge Rates

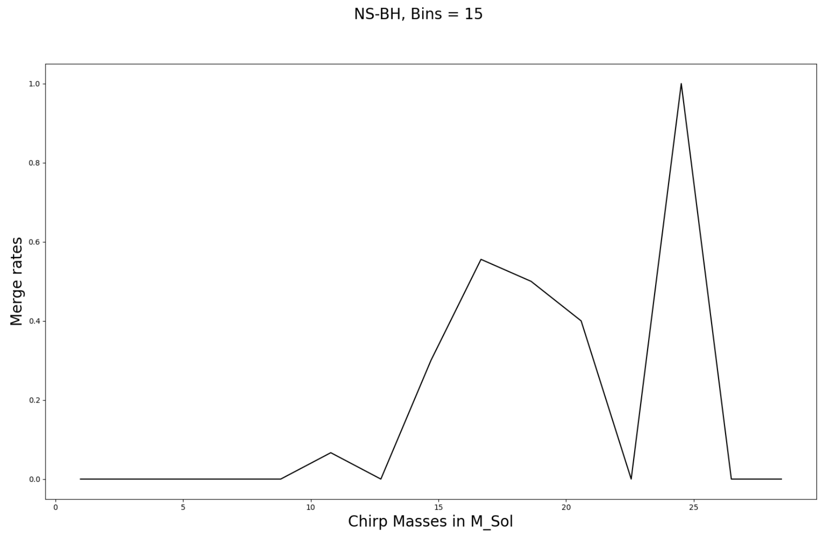

Figure 4.3: Merge rates vs Chirp masses for NS-BH systems

4.4 DCO Merge Rates

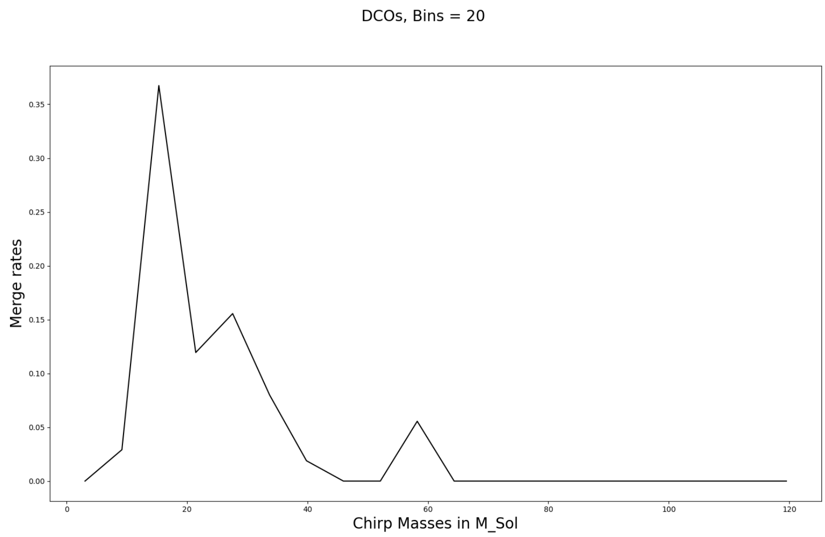

Figure 4.4: Merge rates vs Chirp masses for all DCO systems

Index